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Unit - 3 - Continuous Random Variable

* Task - 1 - Continuous Random Variables.

1 Define (i) Continuous Random Variable (ii) Probability density function.

=> Continuous Random Variable:

A Continuous Random Variable is a random variable that has only continuous values and values are uncountables and related to real numbers.

=> Probability Density Function:

Probability Density Function is used to specify the Probability of the random variable within a particular range of values or Integral of value.

⇒ Probability Density function is given by

$$F(x) = \begin{cases} C(4x - 2x^2), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(i) What is the value of C?
(ii) Find $P(X > 1)$

(i) We know that,

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1$$

So, $$\int_{-\infty}^{a} C(4x - 2x^2) \cdot dx = 1$$

$$\therefore \int_{-\infty}^{0} C(4x - 2x^2) \cdot dx + \int_{0}^{2} C(4x - 2x^2) \cdot dx = 1$$

$$\therefore C\left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_0^2 = 1$$

$$\therefore C\left[\left(\frac{4(2)^2}{2} - \frac{2(2)^3}{3}\right) - 0\right] = 1$$

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$\therefore c(24-16) = 3$

$\therefore c = \frac{3}{8}$

(ii) $P(X > 1) = \int_{-\infty}^{1} f(x) \cdot dx$

$= \int_{-\infty}^{0} f(x) \cdot dx + \int_{0}^{1} f(x) \cdot dx$

$= \int_{0}^{1} \frac{3}{8}(4x - 2x^2) \cdot dx$

$= \frac{3}{8}\left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_0^1$

$= \frac{3}{8}\left[\frac{4}{2} - \frac{2}{3} - 0\right]$

$= \frac{3}{8} \times \frac{4}{3}$

$P(X > 1) = \frac{1}{2}$

2 Continuous Random Variable with PDF given by

$$F(x) = \begin{cases} \lambda e^{-\frac{x}{100}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

(a) Function between 50 and 150 hours

(b) Function fewer than 100 hours.

$\Rightarrow$

(a) $P(50 < x < 150) = \int_{50}^{150} F(x) \cdot dx$

$$= \int_{50}^{150}$$

For Findining the value of x, We know that,

$$\int_{-\infty}^{\infty} F(x) \cdot dx = 1$$

$$\therefore \int_{0}^{\infty} \lambda e^{-\frac{x}{100}} \cdot dx = 1$$

$$\therefore X \left[\frac{-e^{-x/100}}{1/100} \right]_{0}^{\infty} = 1$$

$\therefore \lambda [0 - (-100)] = 1$

$\therefore \lambda = \frac{1}{100}$

(a) $P(50 < x < 150) = \int_{50}^{150} f(x) \cdot dx$

$= \frac{1}{100} \int_{50}^{150} e^{-x/100} \cdot dx$

$= \frac{1}{100} \left[\frac{-e^{-x/100}}{1/100} \right]_{50}^{150}$

$= \frac{1}{100} \times 100 \left[e^{-1/2} - e^{-3/2} \right]$

$= 0.3834$

(b) $P(X < 100) = \int_{0}^{100} f(x) \cdot dx$

$= \frac{1}{100} \int_{0}^{100} e^{-x/100} \cdot dx$

$= \frac{1}{100} \left[\frac{-e^{-x/100}}{1/100} \right]_{0}^{100}$

$$= \left[-e^{-x/100}\right]_0^{100}$$

$$= -e^{-1} - (-e^{0})$$

$$= 0.6321$$

3 Let X be a Continuous random variable,

$$f(x) = \begin{cases} kx, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the value of k

(b) Find $P(1/4 < x \leq 2)$

=> We know that,

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1$$

$$\therefore \int_{-\infty}^{0} f(x) \cdot dx + \int_{0}^{1} f(x) \cdot dx = 1$$

$$\therefore \int_{0}^{1} kx = 1$$

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$\therefore k\left[\frac{x^2}{2}\right]_0^1 = 1$

$\therefore k\left[\frac{1}{2} - 0\right] = 1$

$\therefore \frac{k}{2} = 1$

$\therefore \boxed{k = 2}$

(b) $P(1/4 < x < 2) = \int_{1/4}^{2} f(x) \cdot dx$

$= \int_{1/4}^{2} 2x \cdot dx$

$= 2\left[\frac{x^2}{2}\right]_{1/4}^{2}$

$= \left[4 - \frac{1}{16}\right]$

$= \frac{63}{16}$

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4 Is the function $f(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

density function?

=) For Probability Density Function, We know that,

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1$$

So, $$= \int_{-\infty}^{\infty} e^{-x} \cdot dx$$

$$= \int_{-\infty}^{0} e^{-x} \cdot dx + \int_{0}^{\infty} e^{-x} \cdot dx$$

$$= \int_{0}^{\infty} e^{-x} \cdot dx$$

$$= \left[-e^{-x} \right]_{0}^{\infty}$$

$$= \left[-e^{-(\infty)} - (-e^{-0}) \right]$$

$$= 1$$

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Here, $\int_{-\infty}^{\infty} e^{-x} . dx = 1$.

So, $F(x)$ is a Probability Density Function.

=) Find the Probability of interval (1, 2) and Find the cumulative Probability function F(2).

$$P(1,2) = \int_{1}^{2} e^{-x} . dx$$

$$= \left[-e^{-x} \right]_{1}^{2}$$

$$= -e^{-2} - (-e^{-1})$$

$$= 0.2325$$

$$F(2) = \int_{0}^{2} e^{-x} . dx$$

$$= \left[-e^{-x} \right]_{0}^{2}$$

$$= -e^{-2} - (-e^{-0})$$

$$= 1 - 0.1352$$

$$= 0.865$$

5 If $f(x) = ke^{-|x|}$ represents a probability density function in $-\infty \le x \le \infty$ Find the value of k and the Probability between 0 and 4.

=> We know that,

$$\int_{-\infty}^{\infty} f(x) \cdot dx = 1$$

$$\therefore \int_{-\infty}^{\infty} ke^{-|x|} \cdot dx = 1$$

$$\therefore \int_{-\infty}^{0} ke^{-|x|} \cdot dx + \int_{0}^{\infty} ke^{-|x|} \cdot dx = 1$$

$$\therefore \int_{0}^{\infty} ke^{-|x|} \cdot dx = 1$$

$$\therefore k\left[-e^{-x}\right]_{0}^{\infty} = 1$$

$$\therefore k\left[-e^{-\infty} - (-e^{-0})\right] = 1$$

$$\therefore k\left[1 + \frac{1}{0}\right] = 1$$

$$\therefore k = \frac{1}{2}$$

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$\rightarrow \int_0^4 f(x) \cdot dx = \int_0^4 k e^{-|x|} \cdot dx$

$= k \left[-e^{-x} \right]_0^4$

$= \frac{1}{2} \left[-e^{-4} + e^{-0} \right]$

$= \left(1 - e^{-4} \right) \frac{1}{2}$

* Task : 2 : Expected value and Variance for Continuous Random Variable.

1 Write the definition formula for x : Continuous Random Variable.

(a) Mean : The mean of a Continuous Random Variable can be defined as the weighted average value of the random variable x.

$$\mu = E(x) = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

(b) Moment : Moment of Random variable is a expected value of a specified integer power of the deviation of the Random Variable.

$$\mu_r = \int_{-\infty}^{\infty} x^r \cdot f(x) \cdot dx$$

(c) Variance : Variance of Continuous Random Variable is denoted by Var(x).

$$Var(X) = E(x^2) - (E(x))^2$$

$$= \int_a^b x^2 \cdot f(x) \cdot dx - \left[\int_a^b f(x) \cdot x \, dx\right]^2$$

(d) Standard Deviation : Standard Deviation of Continuous Random Variable is denoted by σ_x

$$\sigma_x = \sqrt{Var(x)}$$

2 Find E(X) when Function of

$$f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow E(X) = \int_0^1 x \cdot f(x) \cdot dx$$

$$= \int_0^1 x \cdot 2x \cdot dx$$

$$= 2\int_0^1 x^2 \cdot dx$$

$$= 2\left[\frac{x^3}{3}\right]_0^1$$

$$E(x) = \frac{2}{3}$$

3 Continuous Random Variable with density function,

$$f(x) = \begin{cases} 2(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean, Variance and standard deviation.

$\Rightarrow$ Mean $E(x) = \int_0^1 x \cdot f(x) \cdot dx$

$$= \int_0^1 2(1-x)x \cdot dx$$

$$= 2\int_0^1 x - x^2 \cdot dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$\text{Mean} = \frac{1}{3}$$

$$\rightarrow E(x)^2 = \int_0^1 x^2 \cdot f(x) \cdot dx$$

$$= \int_0^1 x^2 \cdot 2(1-x) \cdot dx$$

$$= 2\int_0^1 x^2 - x^3 \cdot dx$$

$$= 2\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 2\left[\frac{1}{3} - \frac{1}{4}\right]$$

$$= \frac{1}{3}$$

$$Var(x) = E(x)^2 - \left[E(x)\right]^2$$

$$= \frac{1}{6} - \left(\frac{1}{3}\right)^2$$

$$= \frac{1}{6} - \frac{1}{9}$$

$$= 9-6/54$$

$$Var(X) = \frac{1}{18}$$

→ Standard Deviation

$$S.D. = \sqrt{Var(X)}$$

$$= \sqrt{\frac{1}{18}}$$

$$= 0.2357$$

4 Let X be a random variable with PDF given by

$$F(x) = \begin{cases} cx^2, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the constant c
(b) Find E(X) and Var(X)
(c) Find $P(X \geq \frac{1}{2})$

(a) We know that,

$$\int_{-\infty}^{\infty} F(x) \cdot dx = 1$$

$$\therefore \int_{-1}^{1} c\,x^2 \cdot dx = 1$$

$$\therefore c\left[\frac{x^3}{3}\right]_{-1}^{1} = 1$$

$$\therefore \boxed{c = \frac{3}{2}}$$

(b)

$$E(X) = \int_{-1}^{1} x \cdot f(x) \cdot dx$$

$$= \int_{-1}^{1} x \cdot \frac{3}{2} x^2 \cdot dx$$

$$= \frac{3}{2}\left[\frac{x^4}{4}\right]_{-1}^{1}$$

$$= \frac{3}{2}\left[\frac{1}{4} - \frac{1}{4}\right]$$

$$= 0$$

$$E(x)^2 = \int_{-1}^{1} x^2 \cdot f(x) \cdot dx$$

$$= \int_{-1}^{1} x^2 \cdot \frac{3}{2} x^2 \cdot dx$$

$$= \frac{3}{2}\left[\frac{x^5}{5}\right]_{-1}^{1}$$

$$= \frac{3}{2}\left[\frac{1}{5} + \frac{1}{5}\right]$$

$$= \frac{3}{5}$$

$$Var(x) = E(x)^2 - [E(x)]^2$$

$$= \frac{3}{5}$$

(c) $P(x \geq 12) = \int_{12}^{1} f(x) \cdot dx$

$$= \int_{12}^{1} \frac{3}{2} \cdot x^2 \cdot dx$$

$$= \frac{3}{2}\left[\frac{x^3}{3}\right]_{12}^{1}$$

$$= \frac{3}{2}\left[\frac{1}{3} - \frac{1}{24}\right]$$

$$P(X \geq 12) = \frac{7}{16}$$

5 Let X be a continuous random Variable with PDF

$$f(x) = \begin{cases} 3/x^4, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and Variance of X.

$\Rightarrow$ Mean $E(X) = \int_1^\infty x \cdot f(x) \cdot dx$

$$= \int_1^\infty x \cdot \frac{3}{x^4} \cdot dx$$

$$= 3\left[\frac{-1}{2x^2}\right]_1^\infty$$

$$= 3\left[0 - \left(-\frac{1}{2}\right)\right]$$

$$= \frac{3}{2}$$

$$E(x)^2 = \int_1^\infty x^2 \cdot f(x) \cdot dx$$

$$= \int_1^\infty x^2 \cdot \frac{3}{x^4} \cdot dx$$

$$= 3\left[-\frac{1}{x}\right]_1^\infty$$

$$= 3$$

$\rightarrow$ $Var(x) = E(x)^2 - [E(x)]^2$

$$= 3 - \left(\frac{3}{2}\right)^2$$

$$= 3 - \frac{9}{4}$$

$$= \frac{3}{4}$$

6 Suppose X has pdf given by

$$f(x) = \begin{cases} 3x^2, & 0 \leq x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Find the mean
(b) Find the variance

$\Rightarrow$ Mean $E(x) = \int_0^1 x \cdot f(x) \cdot dx$

$= \int_0^1 x \cdot 3x^2 \cdot dx$

$= 3 \left[\frac{x^4}{4} \right]_0^1$

$= \frac{3}{4}$

$\rightarrow E(x)^2 = \int_0^1 x^2 \cdot f(x) \cdot dx$

$= 3 \int_0^1 x^2 \cdot x^2 \cdot dx$

$= 3 \left[\frac{x^5}{5} \right]_0^1$

$= \frac{3}{5}$

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$$Var(x) = E(x)^2 - [E(x)]^2$$

$$= \frac{3}{5} - \left(\frac{3}{4}\right)^2$$

$$= \frac{3}{5} - \frac{9}{16}$$

$$= \frac{48-45}{80}$$

$$= \frac{3}{80}$$

$$Var(x) = 0.0375$$

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* Task : 3 : The Uniform Random Variable.

1 Define Uniform Random Variable and PDF in $(0, 1)$ and (α, β)
Let X be uniformly distributed over (α, β) find $E[X]$ and $Var(X)$

=> Uniform Random Variable :

If x is a Uniform Random variable on the interval (a, b) then PDF is given by

$$f(x) \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$$

-> $E(x) = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$

$$= \int_{a}^{b} x \cdot \frac{1}{b-a} \cdot dx$$

$$= \frac{1}{b-a} \left(\frac{x^2}{2} \right)_a^b$$

$$E(x) = \frac{1}{2(b-a)} \; b^2 - a^2$$

$$= \frac{(b-a)(b+a)}{2(b-a)}$$

$$E(x) = \frac{b+a}{2}$$

$$\rightarrow \; E(x^2) = \int_a^b x^2 \cdot f(x) \cdot dx$$

$$= \frac{1}{b-a} \int_a^b x^2 \cdot dx$$

$$= \frac{1}{b-a} \left(\frac{x^3}{3} \right)_a^b$$

$$= \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{(b-a)(a^2+ab+b^2)}{3(b-a)}$$

$$= \frac{a^2+ab+b^2}{3}$$

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→ $Var(x) = E(x^2) - (E(x))^2$

$$= \frac{a^2+ab+b^2}{3} - \frac{a^2+2ab+b^2}{4}$$

$$= \frac{4a^2+4ab+4b^2-3a^2-6ab-3b^2}{12}$$

$$= \frac{a^2-2ab+b^2}{12}$$

$$Var(x) = \frac{(a-b)^2}{12}$$

2. If X is Uniformly distrbuted with mean 1 and Variance 4/3 Find $P(X \le 0)$

→ Mean of Uniform Random Variable,

$$E(x) = 1 = \frac{a+b}{2}$$

$$\therefore a+b = 2 \quad - (1)$$

→ Variance of Uniform Random Variable,

$$Var(x) = \frac{4}{3} = \frac{(a-b)^2}{12}$$

$$(a-b)^2 = 16 \quad - \text{②}$$

By eq^n 1 and 2,

$$\therefore (a+b)^2 = 4$$
$$(a-b)^2 = 16$$

$$\therefore a^2 + 2ab + b^2 = 4$$
$$a^2 - 2ab + b^2 = 16$$

$$2a^2 + 2b^2 = 20$$

$$\therefore 2(a^2+b^2)\ 20$$

$$\therefore a^2 + b^2 = 10$$

$$\therefore a^2 + 2ab + b^2 = 4$$

$$\therefore 2ab + 10 = 4$$

$$\therefore 2ab = -6$$

$$\therefore ab = -3$$

Here, we get $a = -1$ or 3
$b = +3$ or 1

$$P(X \leq 0) = \frac{1}{b-a} = \frac{1}{3-(-1)}$$

$P(X \leq 0) = \frac{1}{4}$

3 Subway trains on a certain line run every half hour between mid night and six in the morning What is probability that a man entering the station at a random time during this period will have to wait atleast twenty minutes?

=> Here, Given that,

$$f(x) = \begin{cases} \frac{1}{30}, & 0 < x < 30 \\ 0, & \text{otherwise.} \end{cases}$$

→ For atleast twenty minutes probability,

$$P(x \geq 20) = \int_{20}^{30} f(x) \cdot dx$$

$$= \int_{20}^{30} \frac{1}{30} \cdot dx$$

$$= \frac{1}{30} (x)_{20}^{30}$$

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$$P(x \geq 20) = \frac{30-20}{30} = \frac{1}{3}$$

4 You arrive at bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

(a) Wait longer than 10 Minutes?

(b) Wail longer than 20 Minutes?

=) Here, Given that,

$$f(x) = \begin{cases} 1/30, & 0 < x < 30 \\ 0, & \text{otherwise} \end{cases}$$

a For longer than 10 Minutes.

$$P(X > 10) = \int_{10}^{30} f(x) \cdot dx$$

$$= \int_{10}^{30} \frac{1}{30} \cdot dx$$

$$= \frac{1}{30} \left(x \right)_{10}^{30}$$

$$P(X > 10) = \frac{30-10}{30} = \frac{2}{3}$$

b) train at arrive 10:20 to 10:30

$$P(20 \le x \le 30) = \int_{20}^{30} f(x) \cdot dx$$

$$= \int_{20}^{30} \frac{1}{30} \cdot dx$$

$$= \frac{1}{30} (x)_{20}^{30}$$

$$P(20 \le x \le 30) = \frac{30-20}{30} = \frac{1}{3}$$

5 Bues arrive at a specified stop at 15-minute intervalse starting at 7 P.M. That is they arrive at 7, 7:15, 7:30 and so on. If a Passenger arrives at between 7 and 7:30. Find the Probability that
(a) Wait less than 5 minutes?
(b) More than 10 minutes?

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$\Rightarrow$ Here Given that,

$$F(x) = \begin{cases} 1/15, & 0 < x < 15 \\ 0, & \text{otherwise} \end{cases}$$

a Less than 5 Minutes,

$$P(X < 5) = \int_0^5 F(x) \cdot dx$$

$$= \int_0^5 \frac{1}{15} \cdot dx$$

$$= \frac{1}{15} (x)_0^5 \cdot$$

$$= \frac{5}{15}$$

$$P(x < 5) = \frac{1}{3}$$

b More than 10 Minutes

$$P(X > 10) = \int_{10}^{15} F(x) \cdot dx$$

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$$= \int_{10}^{15} \frac{1}{15} \cdot dx$$

$$= \frac{1}{15} (x)_{10}^{15}$$

$$= \frac{15-10}{15}$$

$$P(x > 10) = \frac{1}{3}$$

6 The continuous random variable X is uniformly distributed over the interval $[-4, 6]$

(a) Mean of X
(b) Find $P(X \leq 2.4)$
(c) $P(-3 < x - 5 < 3)$

Here, Given that

$$f(x) = \begin{cases} 1/10, & -4 < x < 6 \\ 0, & \text{otherwise} \end{cases}$$

(a) Mean

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$E(X) = \int_{-4}^{6} x \cdot \frac{1}{10} \cdot dx$$

$$= \frac{1}{10}\left[\frac{x^2}{2}\right]_{-4}^{6}$$

$$= \frac{20}{20}$$

$$E(X) = 1$$

(b) $$P(x \leq 2.4) = \int_{-4}^{2.4} f(x) \cdot dx$$

$$= \int_{-4}^{2.4} \frac{1}{10} \cdot dx$$

$$= \frac{1}{10}\left[x\right]_{-4}^{2.4}$$

$$= \frac{6.4}{10}$$

$$P(x \leq 2.4) = 0.64$$

(c) $P(-3 < x-5 < 3)$

For Finding the Probability,
We have to add 5

$\therefore P(-3+5 < x-5+5 < 3+5)$

$\therefore P(2 < x < 8)$

$$\therefore P(2 < x < 8) = \int_2^6 f(x) \cdot dx + \int_6^\infty 0 \cdot dx$$

$$= \int_2^6 \frac{1}{10} \cdot dx$$

$$= \frac{1}{10} (x)_2^6$$

$$= \frac{1}{10} (6-2)$$

$$= \frac{4}{10}$$

$P(2 < x < 8) = 0.4$

The continuous random variable Y is uniformly distrubeted over the interval [a, 4a]

(d) Use Integration to show that $E(Y^2) = 7a^2$

(e) Find Var(X)

(f) Given that $P\left(X < \frac{8}{3}\right) = P\left(Y < \frac{8}{3}\right)$

Find value of a.

=) Here, Given that

$$f(x) = \begin{cases} \frac{1}{3a}, & a < x < 4a \\ 0, & \text{otherwise} \end{cases}$$

(d) $E(Y^2) = 7a^2$

$L.H.S = E(Y^2)$

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$$= \int_a^{4a} y^2 \cdot f(y) \cdot dy$$

$$= \int_a^{4a} y^2 \cdot \frac{1}{3a} \cdot dy$$

$$= \frac{1}{3a} \left[\frac{y^3}{3} \right]_a^{4a}$$

$$= \frac{1}{9a} (64a^3 - a^3)$$

$$= \frac{63a^3}{9a}$$

$$= 7a^2$$

L.H.S = R.H.S.

(B)

$$E(y) = \int_a^{4a} y \cdot f(y) \cdot dy$$

$$E(Y) = \int_a^{4a} \frac{1}{3a} \cdot Y \cdot dY$$

$$= \frac{1}{3a}\left[\frac{Y^2}{2}\right]_a^{4a}$$

$$= \frac{16a^2 - a^2}{6a}$$

$$= \frac{5a}{2}$$

$$Var(X) = E(Y^2) - (E(Y))^2$$

$$= 7a^2 - \left(\frac{5a}{2}\right)^2$$

$$= 7a^2 - \frac{25a^2}{4}$$

$$= \frac{28a^2 - 25a^2}{4}$$

$$Var(X) = \frac{3a^2}{4}$$

(F) Given that,

$\cancel{P}$

$$P\left(x < \frac{8}{3}\right) = P\left(Y < \frac{8}{3}\right)$$

$$\therefore \int_{-4}^{8/3} F(x) \cdot dx = \int_{a}^{8/3} F(Y) \cdot dY$$

$$\therefore \int_{-4}^{8/3} \frac{1}{10} \cancel{x} \cdot dy = \int_{a}^{8/3} \frac{1}{3a} \cancel{y} \cdot dY$$

$$\therefore \frac{1}{10}\left[x\right]_{-4}^{8/3} = \frac{1}{3a}\left[Y\right]_{a}^{8/3}$$

$$\therefore \frac{1}{10}\left[\frac{20}{3}\right] = \frac{1}{3a}\left(\frac{8-3a}{3}\right)$$

$$\therefore \frac{2}{3} = \frac{1}{9a}(8-3a)$$

$\therefore \; 6a = 8 - 3a$

$\therefore \; 9a = 8$

$\therefore \; a = \frac{8}{9}$

7 If x is uniformly distributed over (0, 10) calculate the probability that,

(a) $X < 3$
(b) $X > 6$
(c) $3 < x < 8$

Here, Given that,

$$f(x) = \begin{cases} 1/10, & 0 < x < 10 \\ 0, & \text{otherwise} \end{cases}$$

(a) $P(X < 3) = \int_0^3 f(x) \cdot dx$

$$= \int_0^3 \frac{1}{10} \cdot dx$$

$$= \frac{1}{10} (x)_0^3$$

$$P(X<3) = \frac{3}{10}$$

(b) $$P(x>6) = \int_6^{10} f(x) \cdot dx$$

$$= \int_6^{10} \frac{1}{10} \cdot dx$$

$$= \frac{1}{10} [x]_6^{10}$$

$$= \frac{10-6}{10}$$

$$P(x>6) = \frac{4}{10} = \frac{2}{5}$$

(c) $P(3 < x < 8) = \int_3^8 f(x) \cdot dx$

$= \int_3^8 \frac{1}{10} \cdot dx$

$= \frac{1}{10} (x)_3^8$

$= \frac{5}{10}$

$P(3 < x < 8) = \frac{1}{2}$

* Task : 4 : Normal Random Variable

1 Define Normal Random Variable and Probability Density Function Find E(X) and Var(X).

=) Normal Random Variable :

If x is a Normal Random Variable then its PDF is given by

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x - \mu/\sigma)^2}$$

Mean $E(x) = \int_{-\infty}^{\infty} x \cdot F(x) \cdot dx$

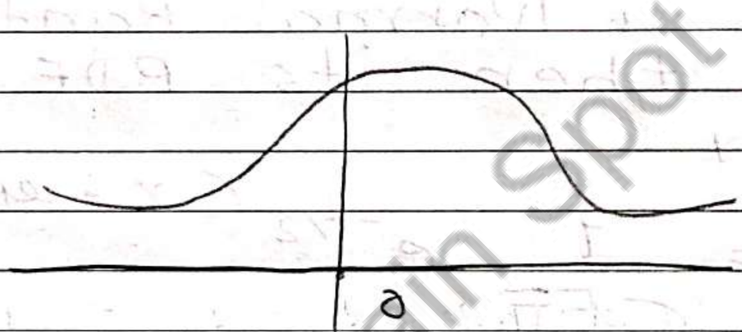
Variance $Var(x) = E(x^2) - (E(x))^2$

2 The IQ's of army volunteers in a given year are normally distributed with mean 110 and standard deviation 10. The army wants to give advanced training

to 20% of those recruits with the highest scores.

Here, Given Mean = 110
Standard Deviation = 10

$P = 20\% = \frac{20}{100} = 0.2$



Here, From the Graph,
Right side of z value is 0.5
So, Value of $Z = 0.5 - 0.2$
$= 0.3$

From the Distruction table
$Z = 0.3$ value is 0.84

$\therefore Z = \frac{x - \mu}{\sigma}$

$\therefore 0.84 = \dfrac{x - 110}{10}$

$\therefore x = 118.4$

3 For a normal distribuction with mean 2 and variance $\sigma^2 = 9$. Find the value of x such that the Probability of the $[2, x]$ is 0.4115

$\Rightarrow$ Here, Given Mean = 2
Variance $\sigma^2 = 9$
Standard Deviation $\sigma = 3$

$\therefore P(2 \le x \le x) = 0.4115$

$\therefore P(X \le x) - P(x \le 2) = 0.4115$

$\therefore P\left(z \le \dfrac{x-\mu}{\sigma}\right) - P(z \le 0) = 0.4115$

$\therefore P\left(z \le \dfrac{x-\mu}{\sigma}\right) - 0.5 = 0.4115$

$$\therefore P\left(z \le \frac{x-\mu}{\sigma}\right) = 0.9115$$

From the Distrubuction table Value of 0.9115 is value of
$z = 1.35$

$$\therefore P\left(z \le \frac{x-\mu}{\sigma}\right) = P(z \le 1.35)$$

$$\therefore \frac{x-\mu}{\sigma} = 1.35$$

$$\therefore x = 1.35(3) + 2$$

$$\therefore x = 6.05$$

4 Most Graduate schools of business application of admission to take the Admission with mean 527 and standard deviation of 112.

(a) Probability to scoring above 500

(b) How High must an individual score on the highest 5% ?

=) Given, Mean = 527
Standard Deviaction = 112

(a)
$$Z = \frac{X - \mu}{\sigma}$$

We have $x = 500$

$$\therefore z = \frac{500 - 527}{112} = -0.24107$$

$P(X > 500) = P(Z > -0.24107)$

$= 1 - P(Z < -0.24)$

From the Distrubuction table $(Z < -0.24)$ value is 0.40317

$P(X > 500) = 1 - 0.40317$

$= 0.5948$

(b) Y = highest mark.

$P(X = Y) = 0.05$

$P(Z > Y) = 0.05$

$\therefore \cancel{1 - P(Z}$

$\therefore P(Z < Y) = 1 - P(Z > Y)$

$= 1 - 0.05$

$= 0.75$

$\therefore$ Value of 0.75 From Distrubuction table is

$Z = 1.643$

$\therefore Z = \frac{x - \mu}{\sigma}$

$\therefore x = 1.643(112) + 527$

$\therefore x = 711.24$

5 The average number of acres burned by forest and range fires is 4300 acres per Year and Standard Deviation

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of 750 acres.

(a) Probability between 2500 and 4200 acres.

(b) What number of burnt acres corresponds to 38th percentile?

=> Here, Given Mean = 4300
Standard Deviation = 750

(a) $P(2500 < x < 4200) =$

$$P\left(\frac{x-\mu}{\sigma} < z < \frac{x-\mu}{\sigma}\right)$$

$$= P\left(\frac{2500-4300}{750} < z < \frac{4200-4300}{750}\right)$$

$$= P(-2.4 < z < -0.1333)$$

$$= P(z < -0.133) - P(z < -2.4)$$

From Distruction table,
$z = -0.133$ value is 0.0082
$z = -2.4$ value is 0.4483

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$= 0.4483 - 0.0082$

$= 0.4401$

(b)
$P(X < x) = 0.38$

$X = 38$ th Percentile

$\therefore P(Z < z) = 0.38$

From Listrubuction table
0.38 is value of $Z = -0.305$

$\therefore Z = -0.305$

$\therefore Z = \dfrac{X - \mu}{\sigma}$

$\therefore x = -0.305(750) + 4300$

$\therefore x = 4071.25$

6 If X is a normal random variable with $\mu = 3$ and $\sigma^2 = 9$ Find

(a) $P(2 < x < 5)$

(b) $P(x > 0)$

(c) $P(|X-3| > 6)$

=) Here, Given that Mean = 3
Standard Deviation = 3

(a) $P(2 < x < 5) =$

$$P\left(\frac{x-\mu}{\sigma} < z < \frac{x-\mu}{\sigma}\right)$$

$$= P\left(\frac{2-3}{3} < z < \frac{5-3}{3}\right)$$

$$= P\left(-\frac{1}{3} < z < \frac{2}{3}\right)$$

$$= P(z < 2/3) - P(z < -1/3)$$

From distrubuction table
$z = 2/3$ value is ~~0.11~~ 0.4899
$z = 1/3$ value is 0.11200

$$P(2 < x < 5) = 0.4899 - 0.11200$$
$$= 0.3779$$

(b) $P(X > 0) = P\left(\frac{x-3}{3} > \frac{0-3}{3}\right)$

$$= P(z > -1)$$

From Distrubuction table
$z = -1$ value is 0.8413

$P(x > 0) = \cancel{0.37}\ 0.8413$

(c) $P(|X-3| > 6) =$

$$P\left(\frac{X-3}{3} > \frac{9-3}{3}\right) + P\left(\frac{x-3}{3} > \frac{-3-5}{2}\right)$$

$$= P(z > 2) + P(z < -2)$$

$$= 0.0456$$

7 Mean is 270 and $\sigma^2 = 100$.
Find the Probability 290 days before and 240 days after days?

=> Here, Given Mean = 270
$\sigma^2 = 100$
Standard Deviation $\sigma = 10$

$P(x > 290) + P(x < 240) =$

$= P\left(\frac{x - 270}{10} > z\right) + P\left(\frac{x - 270}{10} < z\right)$

$= 0.0241$

8 Using Distruction table Find the Probability.

(a) $P(0 \leq x \leq 1.42) =$

$= \phi(1.42) - \phi(0)$

$= 0.922 - 0.500$

$= 0.422$

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(b) $P(-0.73 \leq x \leq 0) = \phi(0) - \phi(-0.73)$

$= \phi(0) - (1 - \phi(0.73))$

$= 0.500 - (1 - 0.7673)$

$= 0.2673$

(c) $P(-1.37 \leq x \leq 2.01)$

$= \phi(2.01) - \phi(-1.37)$

$= \phi(2.01) - (1 - \phi(1.37))$

$= 0.9778 - (1 - 0.9147)$

$= 0.8925$

(d) $P(0.65 \leq x \leq 1.26)$

$= \phi(1.26) - \phi(0.65)$

$= 0.8462 - 0.7422$

$= 0.1540$

(e) $P(-1.79 \leq X \leq -0.54)$

$= \phi(-0.54) - \phi(-1.79)$

$= (1 - \phi(0.54)) - (1 - \phi(1.79))$

$= (1 - 0.7054) - (1 - 0.9633)$

$= 0.2579$

(f) $P(X \geq 1.13) = 1 - \phi(1.13)$

$= 1 - 0.8708$

$= 0.1292$

(g) $P(|x| \leq 0.5) = P(-0.5 \leq x \leq 0.5)$

$= P(0.5) - \phi(-0.5)$

$= P(0.5) - (1 - \phi(0.5))$

$= 0.6915 - (1 - 0.6915)$

$= 0.3830$

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Q The average test marks in Particular class is 79. The standard deviation is 5. If class of 200 did not receive marks between 75 and 82?

=) Here, Given that $\sigma = 5$
$\mu = 79$

So, $Z = \dfrac{x - \mu}{\sigma}$

For $x = 75$, $Z = \dfrac{75 - 79}{5}$

$Z = -0.8$

For $x = 82$, $Z = \dfrac{82 - 79}{5}$

$Z = 0.6$

$P(-0.8 < Z < 0.6) = P(0.6) + P(-0.8)$

$= 0.2881 + 0.2258$

$= 0.5139$

Students does not get marks $= 1 - 0.5139$

$= 0.4862$

For 200 students,

$= 0.4862 \times 200$

$= 97$