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Unit - 4 - Correlation and Regression

* Task : 1 : Correlation

1 If the coefficient of correlation between the two variable X and Y is 0.48. the covariance is 36 and the variance X is 16, Find the standard deviation of Y.

=) Here, Given Coefficient $r = 0.48$
and Covariance $Cov(x, y) = 36$
and Variance of X is 16

$$\therefore \text{So } \sigma_x^2 = 16$$

Standard Deviation $\sigma_x = 4$

We know that,

$$r = \frac{Cov(x, y)}{\sigma_x \cdot \sigma_y}$$

$$\therefore \sigma_y = \frac{\mathrm{Cov}(x, y)}{\sigma_x \cdot r}$$

$$= \frac{36}{4 \times 0.48}$$

$$\therefore \sigma_y = 18.75$$

2 Calculate the Coefficient of Correlation:

X	Y	XY	X^2	Y^2
78	84	6552	6084	7056
36	51	1836	1296	2601
98	91	2918	9604	8281
25	60	1500	625	3600
75	68	5100	5625	4624
82	62	5084	6724	3844
90	86	7740	8100	7396
62	58	3596	3844	3364
65	53	3445	4225	2809
39	67	1833	1621	2209

Here,
X = Marks in Economics.
Y = Marks in Statistics.

From the table.

$\Sigma x = 650$, $\Sigma y = 660$

$\Sigma xy = 45604$, $\Sigma x^2 = 47648$

$\Sigma y^2 = 45784$. $n = 10$

Coefficient of Correlation

$$r = \frac{n\,\Sigma xy - (\Sigma x \cdot \Sigma y)}{\sqrt{n \cdot \Sigma x^2 - (\Sigma x)^2} \cdot \sqrt{n \cdot \Sigma y^2 - (\Sigma y)^2}}$$

$$r = \frac{10(45604) - (650 \cdot 660)}{\sqrt{10(47648)(650)^2} - \sqrt{10(45784)(660)^2}}$$

$r = 0.787$

3 Find the correlation between the serum diastolic blood pressure and Chotesterol.

X	Y	XY	X^2	Y^2
307	80	24560	94249	6400
259	75	19425	67081	5625
341	90	30690	116281	8100
317	74	23458	100489	5476
274	75	20550	75076	5625
416	110	45760	173056	12100
267	70	18690	71289	4900
320	85	27112	102400	7225
274	88	24112	75076	7744
336	78	26206	112896	6084

From the table,

$\Sigma x = 3111$, $\Sigma y = 825$

$\Sigma x^2 = 987893$, $\Sigma y^2 = 69279$

$\Sigma xy = 260653$, $n = 10$

Here,

x = Cholesterol

y = Blood Pressure

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Correlation Coefficient

$$r = \frac{n\Sigma xy - (\Sigma x \cdot \Sigma y)}{\sqrt{n\cdot\Sigma x^2 - (\Sigma x)^2}\cdot\sqrt{n\cdot\Sigma y^2 - (\Sigma y)^2}}$$

$$r = \frac{10(260653) - (3111)(825)}{\sqrt{10\cdot 987893 - (3111)^2}\cdot\sqrt{10\cdot 69279 - (825)^2}}$$

$$\cancel{r = \frac{2606530 - 2566575}{78388.10152 - 686680.5617}}$$

$r = 0.809$

Find the correlation cofficient between the sales and expenes.

Here,
X = Sales
Y = Expenses

X	Y	X^2	Y^2	xY
50	11	2500	121	550
50	13	2500	169	650
55	14	3025	196	770
60	16	3600	256	960
65	16	4225	256	1040
65	15	4225	225	975
65	15	4225	225	975
60	14	3600	196	840
60	13	3600	169	780
50	13	2500	169	650

From the table,

$\Sigma x = 580$, $\Sigma Y = 140$

$\Sigma x^2 = 34000$, $\Sigma Y^2 = 1982$

$\Sigma xY = 8190$, $n = 10$

Correlation coefficient

$$r = \frac{n \cdot \Sigma xY - (\Sigma x \cdot \Sigma Y)}{\sqrt{n \cdot \Sigma x^2 - (\Sigma x)^2} \cdot \sqrt{n \cdot \Sigma Y^2 - (\Sigma Y)^2}}$$

$$r = \frac{10(8190) - (580 \times 140)}{\sqrt{10(34000) - (580)^2} \cdot \sqrt{10(1982) - (140)^2}}$$

$$r = \frac{81900 - 81200}{\sqrt{3600} \cdot \sqrt{220}}$$

$$r = \frac{700}{60 \times 14.83}$$

$$= \frac{700}{889.8}$$

$$r = 0.786$$

5 Find the coefficient of correlation between the Intelligence Ratio and Emotional Ratio from the following data.

Here,

X = Intelligence Ratio

Y = Emotional Ratio

X	Y	X^2	Y^2	XY
105	101	11025	10201	10605
104	103	10816	10609	10712
102	100	10404	10000	10200
101	98	10201	9604	9898
100	95	10000	9025	9500
99	96	9801	9216	9504
98	104	9604	10816	10192
96	92	9216	8468	8832
93	97	8649	9409	9921
92	94	8464	8836	8648

From the table,

$\Sigma x = 990$, $\Sigma y = 980$
$\Sigma x^2 = 98180$, $\Sigma y^2 = 96184$
$\Sigma xy = 97122$, $n = 10$

Coefficient of correlation

$$r = \frac{n\,\Sigma xy - (\Sigma x \cdot \Sigma y)}{\sqrt{n \cdot \Sigma x^2 - (\Sigma x)^2} \cdot \sqrt{n \cdot \Sigma y^2 - (\Sigma y)^2}}$$

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$$r = \frac{10(97122) - (990 \times 980)}{\sqrt{10(98180) - (990)^2} \cdot \sqrt{10(98184)^2 - (980)^2}}$$

$r = 0.651$

6 Calculate the Spearman's rank correlation coefficient

X	Y	R_x	R_y	$d = R_x - R_y$	d^2
39	47	8	10	-2	4
65	53	6	8	-2	4
62	58	7	7	0	0
90	86	2	2	0	0
82	62	3	5	-2	4
75	68	5	4	1	1
25	60	10	6	4	16
98	91	1	1	0	0
36	51	9	9	0	0
78	84	4	3	1	1

Here, X = Advertisement Cost
Y = Sales

From the table,

$$\sum d^2 = 30 \quad , \quad n = 10$$

Spearman's rank correlation coefficient

$$r = \frac{1 - 6\sum d^2}{n(n^2 - 1)}$$

$$r = \frac{1 - 30(6)}{10(10^2 - 1)}$$

$$r = 1 - \frac{180}{990}$$

$$r = 0.82$$

7 Calculate Spearman's rank correlaration coefficient.

Here,

X = One Judges Rank
Y = Second Judges Rank

X	Y	R_x	R_y	$d = R_x - R_y$	d^2
1	12	12	1	11	121
2	9	11	4	7	49
3	6	10	7	3	9
4	10	9	3	6	36
5	3	8	10	-2	4
6	5	7	8	1	1
7	4	6	9	-3	9
8	7	5	6	-1	1
9	8	4	5	-1	1
10	2	3	11	-8	64
11	11	2	2	0	0
12	1	1	12	-11	121

From table, $\Sigma d^2 = \cancel{461}\ 416$, $n = 12$

$$r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6(\cancel{461}\ 416)}{12(12^2-1)}$$

$$r = 1 - 1.4545$$

$$r = -0.4545$$

8 Using Spearman's rank Corelation Method to determine which pair of judges has the nearest approact.

X	Y	Z	R_x	R_y	R_z	d_1	d_1^2	d_2	d_2^2	d_3	d_3^2
1	3	6	10	8	5	2	4	3	9	5	25
6	5	4	5	6	7	-1	1	1	1	-2	4
5	8	9	6	3	2	3	9	2	4	4	16
10	4	8	1	7	3	-6	36	4	16	-2	24
3	7	1	8	4	10	4	16	-6	36	-2	24
2	10	2	9	1	9	8	64	-8	64	0	0
4	2	3	7	9	8	-2	4	1	1	-1	1
9	1	10	2	10	1	-8	64	9	81	1	1
7	6	5	4	5	6	-1	1	-1	1	-2	4
8	9	7	3	2	4	1	1	-2	4	-1	1

Here, $X = 1^{st}$ Judge

$Y = 2^{nd}$ Judge

$Z = 3^{rd}$ Judge

$d_1 = R_x - \cancel{\partial} R_y$

$d_2 = R_y - R_z$

$d_3 = R_x - R_z$

From the table,

$\Sigma d_1^2 = 200$ $\qquad$ $\Sigma d_2^2 = 214$

$\Sigma d_3^2 = 560$ $\qquad$ $n = 10$

-> Corelation Coefficient of X and Y $= 1 - \frac{6\Sigma d_1^2}{n(n^2-1)}$

$$r_1 = 1 - \frac{6(200)}{10(10^2-1)}$$

$$r_1 = 1 - 1.2121$$

$$r_1 = -0.2121$$

-> Corelation Coefficient of Y and Z $r_2 = 1 - \frac{6\Sigma d_2^2}{n(n^2-1)}$

$$r_2 = 1 - \frac{6(214)}{10(10^2-1)}$$

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$r_2 = 1 - 1.2969$

$r_2 = -0.2969$

Corelation coefficient of X and Z

$$r_3 = 1 - \frac{6 \sum d_3^2}{n(n^2 - 1)}$$

$$r_3 = 1 - \frac{6(60)}{10(10^2 - 1)}$$

$r_3 = 1 - 0.3636$

$r_3 = 0.6363$

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* Task : 2 Regression

1 At the time of estimation of the regression equations of the two variables x and Y, the following result were obtained: $\bar{x} = 90$, $\bar{Y} = 70$, $n = 10$, $\Sigma x^2 = 6360$, $\Sigma Y^2 = 2860$, $\Sigma xY = 3900$.
Obtain the regression equations.

=> Here, Given $\bar{x} = 90$, $\bar{Y} = 70$,
$n = 10$, $\Sigma x^2 = 6360$
$\Sigma Y^2 = 2860$, $\Sigma xY = 3900$

We know that,
$$\bar{X} = \frac{\Sigma x}{n}$$
$$\therefore \Sigma x = \bar{X} \cdot n = 90 \times 10$$
$$\therefore \Sigma x = 900$$
$$\therefore \bar{Y} = \frac{\Sigma Y}{n}$$
$$\therefore \Sigma Y = \bar{Y} \cdot n = 70 \times 10$$
$$\therefore \Sigma Y = 700$$

$$b_{\cancel{y}x} = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b_{\cancel{y}x} = \frac{39000 - 630000}{63600 - 810000}$$

$$= \frac{-591000}{-746400}$$

$$b_{\cancel{y}x} = 0.79180$$

$$b_{x\cancel{y}} = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{39000 - 630000}{28600 - 490000}$$

$$= \frac{-591000}{-461400}$$

$$b_{x\cancel{y}} = 1.28084$$

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→ X on Y Regression Line,

$$X - \bar{X} = b_{xy}(Y - \bar{Y})$$

$$X - 90 = 1.280(Y - 70)$$

$$X = 1.280Y - 89.6 + 90$$

$$X = 1.28Y + 0.40$$

→ Y on X Regression Line

$$Y - \bar{Y} = b_{yx}(X - \bar{X})$$

$$Y - 70 = 0.79(X - 90)$$

$$Y = 0.79x - 71.1 + 70$$

$$Y = 0.79x - 1.1$$

2 Obtain the equation of Regression line.

Here, X = Sales
Y = Purchaes

X	Y	X^2	Y^2	XY
91	71	8281	5041	6461
97	75	9409	5625	7275
108	69	11664	4761	7452
121	97	14641	9409	11737
67	70	4489	4900	4690
124	91	15376	8281	11284
51	39	2601	1521	1989
73	61	5329	3721	4453
111	80	12321	6400	8880
57	47	3249	2209	2679

From the table,

$\Sigma x = 900$, $\Sigma Y = 700$

$\Sigma x^2 = 87360$, $\Sigma Y^2 = 51868$

$\Sigma XY = 66900$, $n = 10$

$$\bar{X} = \frac{\Sigma x}{n} = \frac{900}{10} = 90$$

$$\bar{Y} = \frac{\Sigma y}{n} = \frac{700}{10} = 70$$

$\rightarrow$ $b_{yx} = \dfrac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$

$= \dfrac{669000 - 630000}{873600 - 810000}$

$= \dfrac{39000}{63600}$

$= 0.6132$

$\rightarrow$ $b_{xy} = \dfrac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$

$= \dfrac{669000 - 630000}{518680 - 490000}$

$= 1.361$

$\rightarrow$ X on Y Regression Line,

$X - \bar{X} = b_{xy}(Y - \bar{Y})$

$X - 90 = 1.361(Y - 70)$

$X = 1.361Y - 5.27$

→ Y on X Regression Line,

$$Y - \bar{Y} = b_{YX}(x - \bar{x})$$

$$Y - 70 = 0.6132(x - 90)$$

$$Y = 0.6132x + 14.812$$

3 Obtain the line of regression of Y on X,

X	Y	X^2	Y^2	XY
1.53	33.5	2.34	1122.25	51.255
1.78	36.5	3.168	1332.25	64.97
2	40	4	1600	80
2.95	45.8	8.7	2097.64	135.11
3.42	53.5	11.69	2862.25	182.97

Here, X = Rainfall
Y = Discharge

From the table,

$\Sigma x = 11.68$, $\Sigma Y = 209.3$
$\Sigma x^2 = 29.908$, $\Sigma Y^2 = 9014.39$
$\Sigma xy = 514.305$, $n = 5$

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- $\bar{x} = \frac{\Sigma x}{n} = \frac{11.68}{5} = 2.336$

- $\bar{y} = \frac{\Sigma y}{n} = \frac{209.3}{5} = 41.86$

- Y on X Regression Line

$b_{yx} = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$

$= \frac{2571.525 - 2444.62}{149.54 - 136.4224}$

$= \frac{126.901}{13.1176}$

$= 9.674$

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 41.86 = 9.674x - 22.598$

$y = 9.67x \cancel{+ 17.15} - 19.26$

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4 Find the line of Regression of Y on X.

X	Y	X^2	XY
15	12	225	180
15	10	225	150
30	25	900	750
30	21	900	630
45	31	2025	1395
45	33	2025	1485
60	44	3600	2640
60	39	3600	2340

Here, X = Temperature
Y = Chemical Compound

From the table,

$\Sigma x = 300$, $\Sigma x^2 = 13522$
$\Sigma Y = 215$, $\Sigma xY = 9570$
$\cancel{\&}$ $n = 8$

$\rightarrow$ $\bar{X} = \dfrac{\Sigma x}{n} = \dfrac{300}{8} = 37.5$

$$\bar{Y} = \frac{\Sigma y}{n} = \frac{215}{8} = 26.875$$

$$d\cancel{y}x = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{76560 - 64500}{108176 - 90000}$$

$$= \frac{12060}{18176}$$

$$= 0.67$$

- Y on X Regression line,

$$Y - \bar{Y} - d\cancel{y}x(X - \bar{X})$$

$$Y - 26.875 = 0.67(X - 37.5)$$

$$Y = 0.67x - 24.8625 + 26.875$$

$$Y = 0.67x + 1.75$$

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5 The two regression equation of the variable X and Y are $x = 19.13 - 0.87Y$ and $Y = 11.64 - 0.5x$, Find Mean of X and Y and Correlation coefficient between x and Y.

=> Here, Given,

$$X = 19.13 - 0.87Y$$
$$Y = 11.64 - 0.5x$$

From eq^n of x and Y

$$x = 19.13 - 0.87(11.64 - 0.5x)$$

$$x = 19.13 - 10.1268 + 0.435x$$

$$\therefore 0.565\bar{x} = 9.0032$$

$$\therefore \bar{X} = 15.93$$

By eq^n of Y

$$\bar{Y} = 11.64 - 0.5(15.93)$$

$\bar{y} = 11.64 - 7.965$

$\bar{y} = 3.675$

-> For Correlation coefficient,

By eq^n of x and y,

We get $b_{xy} = -0.87$
$b_{yx} = -0.5$

$\left(\because \text{Compare with } x = 19.13 - b_{xy} \cdot y \text{ and } y = 11.64 - b_{yx} x \; eq^n \right)$

$r = \sqrt{b_{xy} \cdot b_{yx}}$

$= \sqrt{(-0.87)(-0.5)}$

$r = 0.6595$

6 Estimate the monthly sales when the company will spent 50000 on advertisement, if the data on Y and X as follows.

X	Y	X^2	XY
73	43	5476	3182
76	44	5776	3344
60	36	3660	2160
68	38	4624	2584
79	47	6241	3713
70	40	4900	2800
71	41	5041	2911
94	54	8836	5076

Here, X = Monthly Sales
Y = Cost

From the table,

$\Sigma x = 592$, $\Sigma x^2 = 44494$
$\Sigma y = 343$, $\Sigma xy = 25770$
$n = 8$

$$\bar{X} = \frac{\Sigma x}{n} = \frac{592}{8} = 74$$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{343}{8} = 42.876$$

→ $d_{xy} = \dfrac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$

$\cancel{= 25/77}$

$= \dfrac{206160 - 203056}{355952 - 350464}$

$= \dfrac{3104}{5488}$

$= 0.565$

→ Y on X Regression line,

$y - \bar{y} = d_{\cancel{x}x}(x - \bar{x})$

$y - 42.876 = 0.565(x - 74)$

$y = 0.565x - 41.81 + 42.876$

$y = 0.565x + 1.066$

→ For $X = 50,000$,

$$Y = 0.565(50,000) + 1.066$$

$$Y = 28251.066$$

7 Obtain the line of Regression of Y on X for the following data.

X	Y	X^2	XY
66	145	4356	9570
38	124	1444	4712
56	147	3136	8232
42	125	1764	5250
72	160	5184	11520
36	118	1296	4248
63	149	3969	9387
47	128	2209	6016
55	150	3025	8250
45	124	2025	5580

Here,
X = Age
Y = Blood Pressure

From the table,

$\Sigma x = 520$, $\Sigma x^2 = 28408$

$\Sigma Y = 1370$, $\Sigma xY = 72765$

$n = 10$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{520}{10} = 52$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{1370}{10} = 137$$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \cdot \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{727650 - 712400}{284080 - 270400}$$

$$= \frac{15250}{13680}$$

$$= 1.1147$$

→ Y on X Regression line.

$$Y - \bar{Y} = d_{yx}(X - \bar{X})$$

$$Y - 137 = 1.114(X - 52)$$

$$Y = 1.114x + 79.0321$$

→ For $X = 50$

$$Y = 1.114(50) + 79.0321$$

$$= 134.7304$$

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Unit - 3 - Continuous Random Variable

* Task : 3 : The Uniform Random Variable.

6 The continuous random variable X is uniformly distributed over the interval [-4, 6]

(a) Mean of X
(b) Find $P(X \le 2.4)$
(c) $P(-3 < x - 5 < 3)$

Here, Given that

$$f(x) = \begin{cases} 1/10, & -4 < x < 6 \\ 0, & \text{otherwise} \end{cases}$$

(a) Mean

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x) \cdot dx$$

$$E(X) = \int_{-4}^{6} x \cdot \frac{1}{10} \cdot dx$$

$$= \frac{1}{10}\left[\frac{x^2}{2}\right]_{-4}^{6}$$

$$= \frac{20}{20}$$

$$E(X) = 1$$

(b) $$P(x \leq 2.4) = \int_{-4}^{2.4} f(x) \cdot dx$$

$$= \int_{-4}^{2.4} \frac{1}{10} \cdot dx$$

$$= \frac{1}{10}\left[x\right]_{-4}^{2.4}$$

$$= \frac{6.4}{10}$$

$$P(x \leq 2.4) = 0.64$$

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$P(-3 < x-5 < 3)$

For Finding the Probability. We have to add 5

$\therefore P(-3+5 < x-5+5 < 3+5)$

$\therefore P(2 < x < 8)$

$$\therefore P(2<x<8) = \int_2^6 f(x)\cdot dx + \int_6^\infty 0 \cdot dx$$

$$= \int_2^6 \frac{1}{10} \cdot dx$$

$$= \frac{1}{10}(x)_2^6$$

$$= \frac{1}{10}(6-2)$$

$$= \frac{4}{10}$$

$P(2<x<8) = 0.4$

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The continuous random variable Y is uniformly distrubeted over the interval [a, 4a]

(d) Use Integration to show that $E(Y^2) = 7a^2$

(e) Find Var(Y)

(f) Given that $P\left(Y < \frac{8}{3}\right) = P\left(Y < \frac{8}{3}\right)$

Find value of a.

=) Here, Given that

$$F(x) = \begin{cases} \frac{1}{3a}, & a < x < 4a \\ 0, & \text{otherwise} \end{cases}$$

(d) $E(Y^2) = 7a^2$

L.H.S. $= E(Y^2)$

$$= \int_a^{4a} y^2 \cdot f(y) \cdot dy$$

$$= \int_a^{4a} y^2 \cdot \frac{1}{3a} \cdot dy$$

$$= \frac{1}{3a} \left[\frac{y^3}{3} \right]_a^{4a}$$

$$= \frac{1}{9a} (64a^3 - a^3)$$

$$= \frac{63a^3}{9a}$$

$$= 7a^2$$

L.H.S = R.H.S.

(e)

$$E(Y) = \int_a^{4a} y \cdot f(y) \cdot dy$$

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$$E(Y) = \int_{a}^{4a} \frac{1}{3a} \cdot y \cdot dy$$

$$= \frac{1}{3a}\left[\frac{y^2}{2}\right]_a^{4a}$$

$$= \frac{16a^2 - a^2}{6a}$$

$$= \frac{5a}{2}$$

$$Var(X) = E(Y^2) - (E(Y))^2$$

$$= 7a^2 - \left(\frac{5a}{2}\right)^2$$

$$= 7a^2 - \frac{25a^2}{4}$$

$$= \frac{28a^2 - 25a^2}{4}$$

$$Var(X) = \frac{3a^2}{4}$$

Given that,

$\cancel{P}$

$$P\left(x < \frac{8}{3}\right) = P\left(Y < \frac{8}{3}\right)$$

$$\therefore \int_{-4}^{8/3} F(x)\cdot dx = \int_{a}^{8/3} F(Y)\cdot dY$$

$$\therefore \int_{-4}^{8/3} \frac{1}{10}\,\cancel{x}\cdot dy = \int_{a}^{8/3} \frac{1}{3a}\,\cancel{y}\cdot dY$$

$$\therefore \frac{1}{10}\left[x\right]_{-4}^{8/3} = \frac{1}{3a}\left[Y\right]_{a}^{8/3}$$

$$\therefore \frac{1}{10}\left[\frac{20}{3}\right] = \frac{1}{3a}\left(\frac{8-3a}{3}\right)$$

$$\therefore \frac{2}{3} = \frac{1}{9a}(8-3a)$$

$\therefore\ 6a = 8 - 3a$

$\therefore\ 9a = 8$

$\therefore\ a = \frac{8}{9}$

7 If x is uniformly distributed over (0, 10) calculate the probability that,

(a) $X < 3$

(b) $X > 6$

(c) $3 < x < 8$

Here, Given that,

$$f(x) = \begin{cases} 1/10, & 0 < x < 10 \\ 0, & \text{otherwise} \end{cases}$$

(a) $P(X < 3) = \int_0^3 f(x) \cdot dx$

$$= \int_0^3 \frac{1}{10} \cdot dx$$

$$= \frac{1}{10} (x)_0^3$$

$$P(X<3) = \frac{3}{10}$$

(b) $$P(x>6) = \int_6^{10} f(x) \cdot dx$$

$$= \int_6^{10} \frac{1}{10} \cdot dx$$

$$= \frac{1}{10} [x]_6^{10}$$

$$= \frac{10-6}{10}$$

$$P(x>6) = \frac{4}{10} = \frac{2}{5}$$

(c) $P(3 < x < 8) = \int_3^8 f(x) \cdot dx$

$$= \int_3^8 \frac{1}{10} \cdot dx$$

$$= \frac{1}{10} (x)_3^8$$

$$= \frac{5}{10}$$

$P(3 < x < 8) = \frac{1}{2}$

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* Task : 4 : Normal Random Variable

1 Define Normal Random Variable and Probability Density Function Find E(X) and Var(X).

=> Normal Random Variable :

If x is a Normal Random variable then its PDF is given by

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(x - \mu/\sigma)^2}$$

Mean $E(x) = \int_{-\infty}^{\infty} x \cdot F(x) \cdot dx$

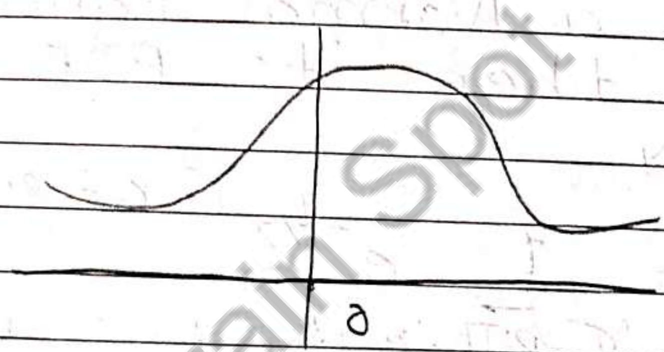
Variance $Var(x) = E(x^2) - (E(x))^2$

2 The IQ's of army volunteers in a given year are normally distributed with mean 110 and standard deviation 10. The army wants to give advanced training

to 20% of those recruits with the highest scores.

=> Here, Given Mean = 110
Standard Deviation = 10

$$P = 20\% = \frac{20}{100} = 0.2$$



Here, From the Graph,
Right side of z value is 0.5
So, Value of Z = 0.5 - 0.2
= 0.3

From the Distruction table
Z = 0.3 value is 0.84

$$\therefore Z = \frac{x - \mu}{\sigma}$$

$\therefore 0.84 = \dfrac{x - 110}{10}$

$\therefore x = 118.4$

3 For a normal distribuction with mean 2 and variance $\sigma^2 = 9$. Find the value of x such that the Probability of the $[2, x]$ is 0.4115

=) Here, Given Mean = 2
Variance $\sigma^2 = 9$
Standard Deviation $\sigma = 3$

$\therefore P(2 \leq x \leq x) = 0.4115$

$\therefore P(X \leq x) - P(x \leq 2) = 0.4115$

$\therefore P\left(Z \leq \dfrac{x - \mu}{\sigma}\right) - P(Z \leq 0) = 0.4115$

$\therefore P\left(Z \leq \dfrac{x - \mu}{\sigma}\right) - 0.5 = 0.4115$

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$$\therefore P\left(z \leq \frac{x-\mu}{\sigma}\right) = 0.9115$$

From the Distrubuction table
Value of 0.9115 is value of
$z = 1.35$

$$\therefore P\left(z \leq \frac{x-\mu}{\sigma}\right) = P(z \leq 1.35)$$

$$\therefore \frac{x-\mu}{\sigma} = 1.35$$

$$\therefore x = 1.35(3) + 2$$

$$\therefore x = 6.05$$

Most Graduate schools of business application of admission to take the Admission with mean 527 and standard deviation of 112.
Probability to scoring above 500
How High must an individual score on the highest 5% ?

=) Given, Mean = 527
Standard Deviaction = 112

(a) $$Z = \frac{X - \mu}{\sigma}$$

We have $x = 500$

$$\therefore z = \frac{500 - 527}{112} = -0.24107$$

$P(X > 500) = P(Z > -0.24107)$

$= 1 - P(Z < -0.24)$

From the Distrubuction table
$(Z < -0.24)$ value is 0.40317

$P(X > 500) = 1 - 0.40317$

$= 0.5948$

(b) Y = highest mark.

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$P(X = Y) = 0.05$

$P(Z > Y) = 0.05$

$\therefore$ ~~1 - P(Z~~

$\therefore P(Z < Y) = 1 - P(Z > Y)$
$= 1 - 0.05$
$= 0.75$

$\therefore$ Value of 0.75 From Distrubuction table is
$Z = 1.643$

$\therefore Z = \dfrac{x - \mu}{\sigma}$

$\therefore x = 1.643(112) + 527$

$\therefore x = 711.24$

5 The average number of acres burned by forest and range fires is 4300 acres per Year and Standard Deviation

of 750 acres.

(a) Probability between 2500 and 4200 acres.

(b) What number of burnt acres corresponds to 38th percentile?

=> Here, Given Mean = 4300
Standard Deviation = 750

(a) $P(2500 < x < 4200) =$

$$P\left(\frac{x-\mu}{\sigma} < z < \frac{x-\mu}{\sigma}\right)$$

$$= P\left(\frac{2500-4300}{750} < z < \frac{4200-4300}{750}\right)$$

$$= P(-2.4 < z < -0.1333)$$

$$= P(z < -0.133) - P(z < -2.4)$$

From Distruction table,
$z = -0.133$ value is 0.0082
$z = -2.4$ value is 0.4483

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$= 0.4483 - 0.0082$

$= 0.4401$

(b)

$P(X < x) = 0.38$

$X = 38$ th Percentile

$\therefore P(z < Z) = 0.38$

From Listrubuction table
0.38 is value of $Z = -0.305$

$\therefore Z = -0.305$

$\therefore Z = \dfrac{X - \mu}{\sigma}$

$\therefore x = -0.305(750) + 4300$

$\therefore x = 4071.25$

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6 If X is a normal random variable with $\mu = 3$ and $\sigma^2 = 9$ Find

(a) $P(2 < x < 5)$

(b) $P(x > 0)$

(c) $P(|X-3| > 6)$

=) Here, Given that, Mean = 3

Standard Deviation = 3

(a) $P(2 < x < 5) =$

$$P\left(\frac{x-\mu}{\sigma} < z < \frac{x-\mu}{\sigma}\right)$$

$$= P\left(\frac{2-3}{3} < z < \frac{5-3}{3}\right)$$

$$= P\left(-\frac{1}{3} < z < \frac{2}{3}\right)$$

$$= P(z < 2/3) - P(z < -1/3)$$

From distrubuction table

$z = 2/3$ value is ~~0.11~~ 0.4899

$z = 1/3$ value is 0.11200

$P(2 < x < 5) = 0.4899 - 0.11200$
$= 0.3779$

$P(x > 0) = P\left(\frac{x-3}{3} > \frac{0-3}{3}\right)$

$= P(z > -1)$

From Distrubuction table
$z = -1$ value is 0.8413

$P(x > 0) = \cancel{0.37}\ 0.8413$

$P(|x-3| > 6) =$

$P\left(\frac{x-3}{3} > \frac{9-3}{3}\right) + P\left(\frac{x-3}{3} > \frac{-3-5}{2}\right)$

$= P(z > 2) + P(z < -2)$

$= 0.0456$

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7 Mean is 270 and $\sigma^2 = 100$.
Find the Probability 290 days before and 240 days after days?

=> Here, Given Mean = 270

$\sigma^2 = 100$

Standard Deviation $\sigma = 10$

$P(x > 290) + P(x < 240) =$

$= P\left(\frac{x-270}{10} > z\right) + P\left(\frac{x-270}{10} < z\right)$

$= 0.0241$

8 Using Distruction table Find the Probability.

(a) $P(0 \leq x \leq 1.42) =$

$= \phi(1.42) - \phi(0)$

$= 0.922 - 0.500$

$= 0.422$

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(b) $P(-0.73 \le x \le 0) = \phi(0) - \phi(-0.73)$

$= \phi(0) - (1 - \phi(0.73))$

$= 0.500 - (1 - 0.7673)$

$= 0.2673$

(c) $P(-1.37 \le x \le 2.01)$

$= \phi(2.01) - \phi(-1.37)$

$= \phi(2.01) - (1 - \phi(1.37))$

$= 0.9778 - (1 - 0.9147)$

$= 0.8925$

(d) $P(0.65 \le x \le 1.26)$

$= \phi(1.26) - \phi(0.65)$

$= 0.8462 - 0.7422$

$= 0.1540$

(e) $P(-1.79 \leq X \leq -0.54)$

$= \phi(-0.54) - \phi(-1.79)$

$= (1 - \phi(0.54)) - (1 - \phi(1.79))$

$= (1 - 0.7054) - (1 - 0.9633)$

$= 0.2579$

(f) $P(X \geq 1.13) = 1 - \phi(1.13)$

$= 1 - 0.8708$

$= 0.1292$

(g) $P(|x| \leq 0.5) = P(-0.5 \leq x \leq 0.5)$

$= P(0.5) - \phi(-0.5)$

$= P(0.5) - (1 - \phi(0.5))$

$= 0.6915 - (1 - 0.6915)$

$= 0.3830$

The average test marks in Particular class is 79. The standard deviation is 5. If class of 200 did not receive marks between 75 and 82?

Here, Given that $\sigma = 5$

$\mu = 79$

So, $z = \dfrac{x - \mu}{\sigma}$

For $x = 75$, $z = \dfrac{75 - 79}{5}$

$z = -0.8$

For $x = 82$, $z = \dfrac{82 - 79}{5}$

$z = 0.6$

$P(-0.8 < z < 0.6) = P(0.6) + P(-0.8)$

$= 0.2881 + 0.2258$

$= 0.5139$

Students does not get marks $= 1 - 0.5139$

$= 0.4862$

For 200 Students,

$= 0.4862 \times 200$

$= 97$