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* Task 1: Cartesian Curves

Trace the following curve.

2 $xy^2 = a^2(a-x)$

Given eq^n of curve is $xy^2 = a^2(a-x)$

1 Symmetry:

In given eq^n of curve we replasing Y by -Y and eq^n remain unchange then curve has symmetry above X axis.

2 Curve passing through the origin.

In eq^n of curve we can see a constant then curve does not passing through the Origin.

3 Intersection with axis.

In eq^n of curve we putting $Y=0$ then we get cordient point Intersection with X axis.

$$\therefore xy^2 = a^2(a-x)$$

$$\therefore 0 = a^3 - a^2x$$

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$$\therefore \boxed{x = a}$$

4 Asympototes

① parallel to x axis :

$$\therefore xy^2 = a^3 - a^2x$$

$$\therefore xy^2 + a^2x = a^3$$

$$\therefore x = \frac{a^3}{y^2+a^2}$$

$$\therefore y^2 + a^2 = 0$$

$$\therefore y = \pm\sqrt{-a}$$

② parallel to Y axis

$$\therefore xy^2 = a^3 - a^2x$$

$$\therefore \boxed{x = 0}$$

5 Region, For region Form the curve eq^n

$$\therefore xy^2 = a^2(a-x)$$

$$\therefore y^2 = \frac{a^2(a-x)}{x}$$

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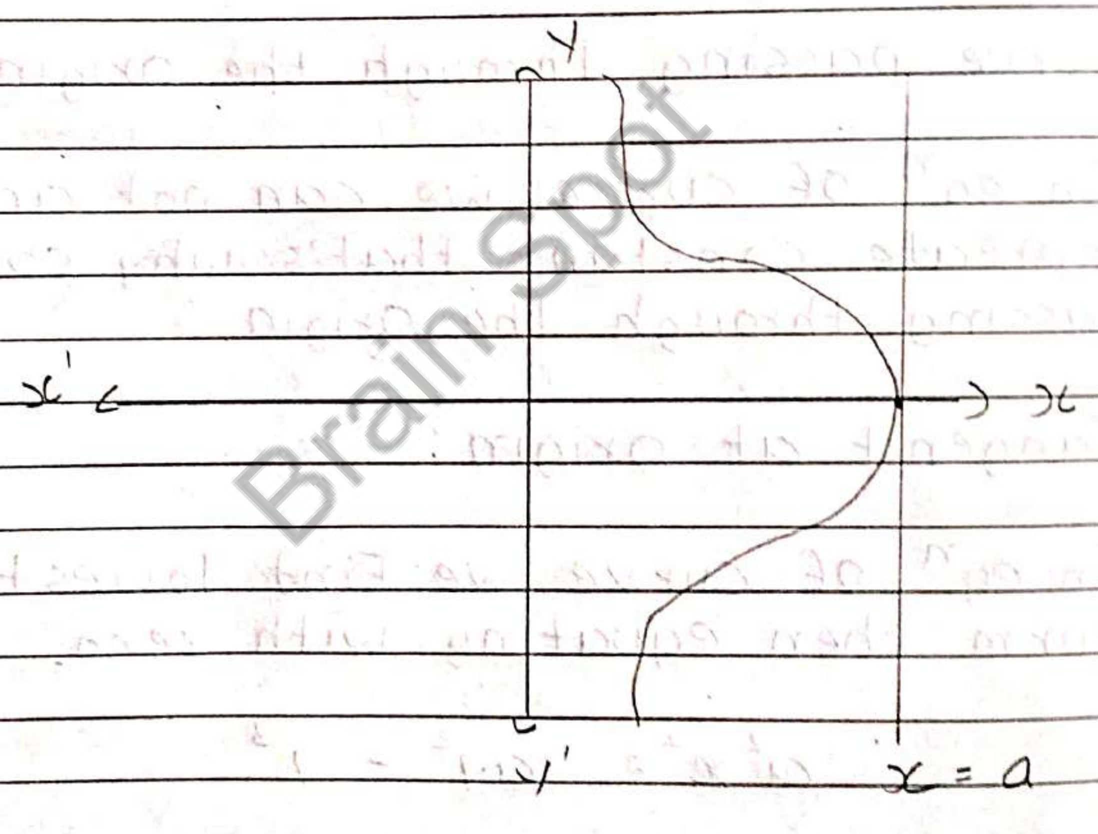
$$\therefore\ y = a\sqrt{\frac{a}{x} - 1}$$

Here, y become imaginery For $x > a$, $x \le 0$.

So, curve does not lies for $x > a$ and $x \le 0$.



3 $a^2x^2 = y^2(2a-y)$

Given eq^n of curve $a^2x^2 = y^2(2a-y)$ — (1)

1 Symmetry:

In eq^n of curve we replassing x by $-x$ eq^n of curve remain unchange then curve has symmetry above y-axis.

2 Curve passing Through the origin

In eq^n of curve we can not any seprecate constant that's why curve passing through the origin.

3 Tangent at origin:

In eq^n of curve we Find lowest degree turm then equating with zero.

$$a^2x^2 = 2ay^2 - y^3$$

$$\therefore a^2x^2 - 2ay^2 = 0$$

$$\therefore a^2x^2 = 2ay^2$$

$$\therefore ax^2 = 2y^2$$

$$\therefore \pm\sqrt{a}x = \pm\sqrt{2}y$$

So, $\sqrt{ax} - \sqrt{2y} = 0$, $-\sqrt{ax} - \sqrt{2y} = 0$
$\sqrt{ax} + \sqrt{2y} = 0$, $-\sqrt{ax} + \sqrt{2y} = 0$

$\therefore \sqrt{ax} - \sqrt{2y} = 0$ and $\sqrt{ax} + \sqrt{2y}$ is tangent at origin.

Here, we get 2 tangent at Origin but tangent is not equal. So, Origin is a become Node.

4 Intersection with axis

(1) For X axis : We take $Y = 0$ In eq^n of curve.

$$\therefore a^2x^2 = y^2(2a - y)$$

$$\therefore a^2x^2 = 0$$

$$\therefore x = 0$$

(2) For Y axis : We take $X = 0$ in eq^n of curve.

$$\therefore a^2x^2 = y^2(2a - y)$$

$$y^2(2a - y) = 0$$

$$\therefore y^2 2a = y^3$$

$\therefore Y = 2a$

Curve intersection with X axis at $x = 0$ and Y axis at $Y = 2a$ point.

5 Region.

$\therefore a^2x^2 = Y^2(2a - Y)$

$\therefore a^2x^2 = Y^2 2a - Y^3$

$\therefore x^2 = \frac{Y^2 2a - Y^3}{a^2}$

$\therefore x = \frac{Y}{a}\sqrt{2a - Y}$

Here, X become imaginery for $Y > 2a$.
So, Curve does not lies for $Y > 2a$.

6 Asympotates.

① parellel to x axis :

In eq^n of curve equate higest degree x coefficient with zero

$\therefore x^2a^2 = 0$

$\therefore a^2 = 0$

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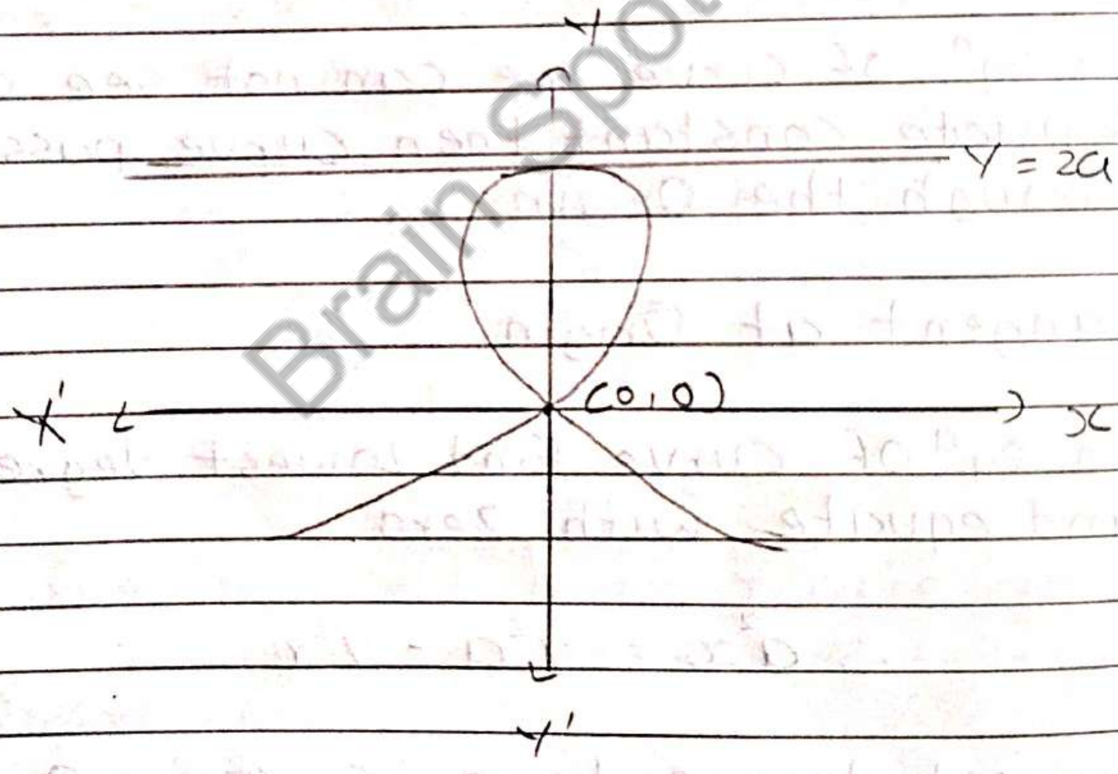
② parellel to Y axis :

In eq^n of curve equate higest degree Y cofficient with Zero.

$$\therefore -y^3 = -1$$
$$So, -1 \neq 0$$

Asympotates is does not exist at parellel to Y axis and X axis.



4 $\cancel{ay^2 =}\ a^2x = y^2(a-x)$

Given eq^n of curve $a^2x = y^2(a-x)$ — (1)

1 Symmetry:

In eq^n of curve we replasing y by $-y$ then eq^n of curve remain unchange then curve has symmetry above x axis.

2 Curve passing through the Origin.

In eq^n of curve we can not see any seprete ~~Seprriete~~ constant then curve passing through the Origin.

3 Tangent at Origin.

In eq^n of curve Find Lowest degree turm and equate with zero.

$$\therefore a^2x = y^2a - y^2x$$

Lowest degree turm is $a^2x = 0$

$$\therefore x = 0$$

4 Intersection with axis.

① For X axis : In eq^n of curve we take $Y=0$

$$\therefore a^2x = Y^2(a-x)$$

$$\therefore x = 0$$

Curve intersect with x axis at Origin

② For Y axis : In eq^n of curve we take $X=0$

$$\therefore a^2x = Y^2(a-x)$$

$$\therefore Y^2a = 0$$

$$\therefore Y = 0$$

Curve intersect with Y axis at Origin

5 Region :

$$\therefore a^2x = Y^2(a-x)$$

$$\therefore Y^2 = \frac{a^2x}{(a-x)}$$

$$\therefore Y = a\sqrt{\frac{x}{a-x}}$$

y become imaginary For $x = a$, $x > a$, $x < 0$.

6 Asympotototes :-

1 parellel to x axis :

In eq^n of curve equate highest degree x coefficent with zero.

$$a^2 x = y^2 a - y^2 x$$

$$\therefore a^2 x + y^2 x = y^2 a$$

$$\therefore x = \frac{y^2 a}{a^2 + y^2}$$

$$\therefore a^2 + y^2 = 0$$

$$\therefore y = \pm \sqrt{-a^2}$$

2 parallel to y axis :

In eq^n of curve equate Highest degree y coeffient with zero.

$$a^2 x = y^2 (a - x)$$

$$\therefore a - x = 0$$

$$\therefore x = a$$

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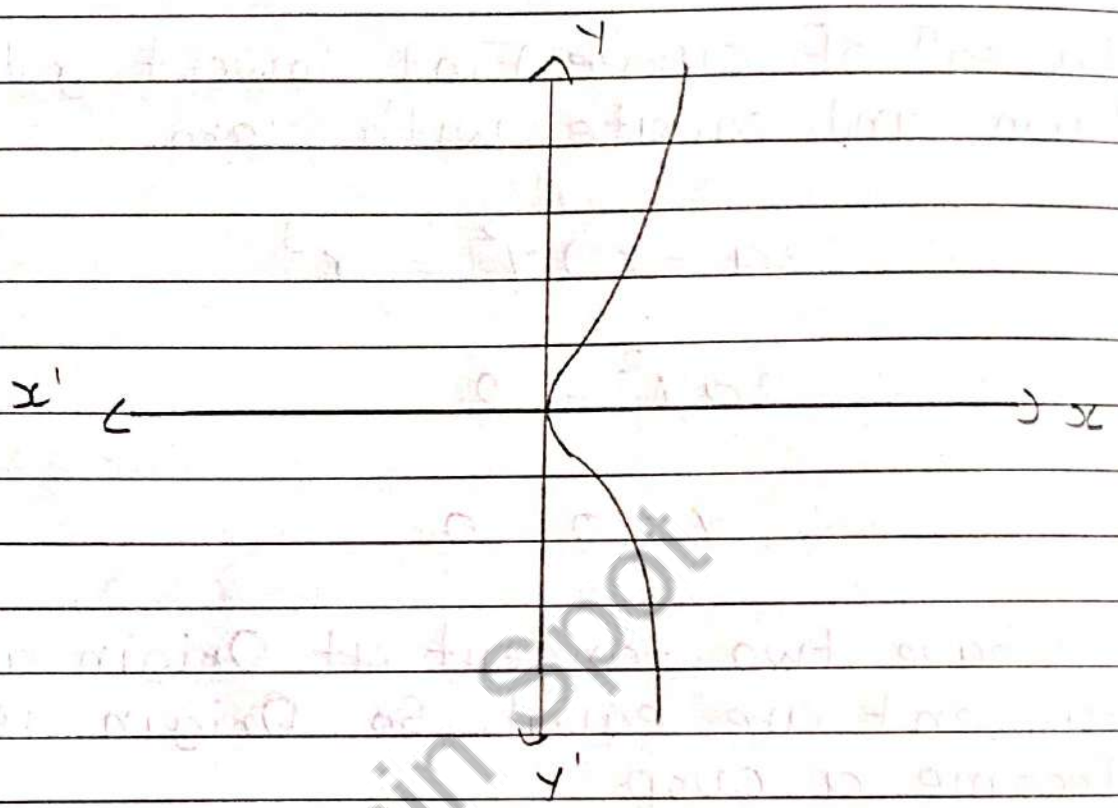
Asympototes is not exist at parellel to x axis. It is exist at parelle to Y axis.



1 $(2a-x)y^2 = x^3$

Given eq^n of curve $(2a-x)y^2 = x^3$ — (1)

1 Symmetry:

In given eq^n of curve we replasing Y by -Y then eq^n remain unchange. So, Curve has Symmetry above X axis.

2 Tangent at Origin:

In given eq^n of curve we can see that any constant turm is not seprete in eq^n. So, Curve has tangent at origin.

3 Tangent at Origin.

In eq^n of curve Find lowest g degree term and equate with zero.

$$\therefore (2a - x) y^2 = x^3$$

$$\therefore 2ay^2 = 0$$

$$\therefore y = 0, 0$$

We have two tangent at Origin and tangent are equal. So, Origin is become a cusp.

4 Intersection with axis

① For X axis : In eq^n of curve putting $y=0$

$$\therefore (2a - x) y^2 = x^3$$

$$\therefore x^3 = 0$$

$$\therefore x = 0$$

② For Y axis : In eq^n of curve putting $X = 0$.

$$\therefore (2a-x)y^2 = x^3$$

$$\therefore 2ay^2 = 0$$

$$\therefore y = 0$$

Intersection with axis curve has Intersect with Y and X axis at origin.

5 Region :

$$(2a-x)y^2 = x^3$$

$$\therefore y^2 = \frac{x^3}{2a-x}$$

$$\therefore y = x\sqrt{\frac{x}{2a-x}}$$

Y become imegnary for $x > 2a$.

6 Asympototes :

(1) parellel to X axis : In eq^n of curve equate higest degree x coeffient with x 0

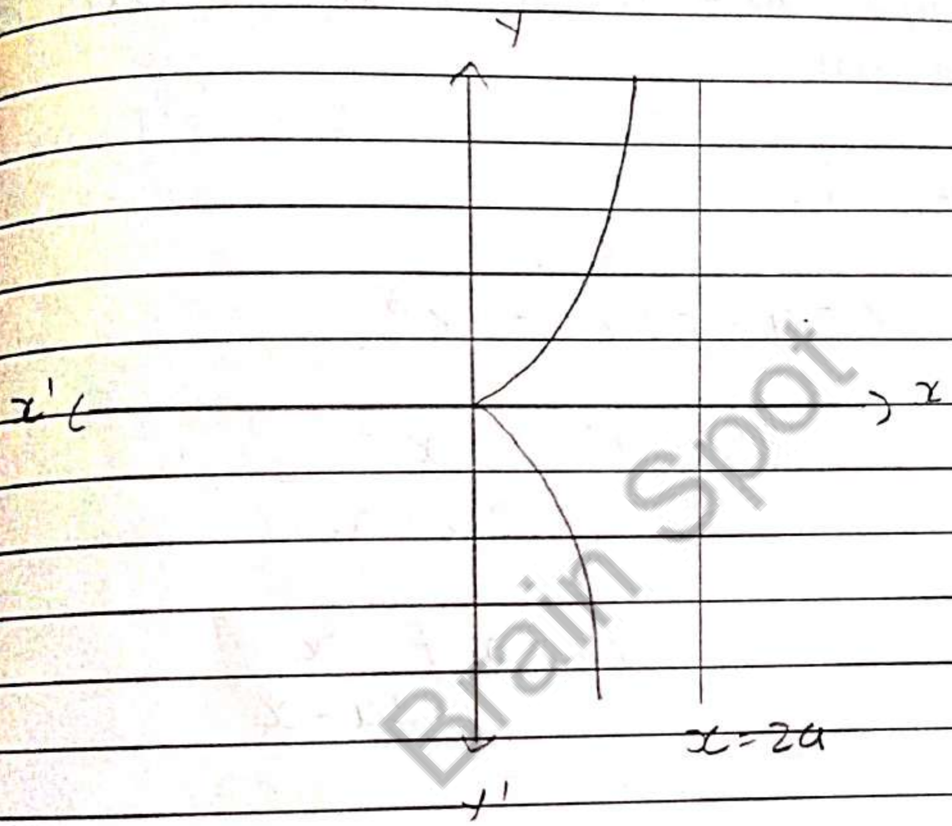
$$\therefore x^3 = 0$$

$$\therefore x = 0$$

(2) parellel to Y axis : In eq^n of curve equate highest degree Y coefficent with 0

$$\therefore 2a - x = 0$$

$$\therefore x = 2a$$



5 $9ay^2 = x(x-3a)^2$

Given eq^n of curve $9ay^2 = x(x-3a)^2$ — (1)

1 Symmetry : In given eq^n of curve we replasing Y by −Y then eq^n of curve remain unchange.
So, Curve has symmetry above X axis.

2 Curve passing through the Origin.

In eq^n of curve we can see that any constant turm does not seprete In eq^n. So, Curve passing through the Origin.

3 Tangent at Origin:

In eq^n of curve Find lowest degree turm and equate with zero.

$$\therefore 9ay^2 = x(x^2 - 6xa + 9a^2)$$

$$\therefore 9ay^2 - x^3 + 6x^2a - 9a^2x = 0$$

here, lowest degree turm

$$\therefore -9a^2x = 0$$

$$\therefore x = 0$$

Curve has tangent at Origin.

4 ~~Intersection with axis~~. Asympototes

① For X axis: In eq^n of curve equate highest degree X coeffient with zero.

$$\therefore x^3 = 0$$

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$x = 0$

② For Y axis : In eq^n of curve equate highest y coeffient with 0

$\therefore -9ay^2 = 0$

$\therefore Y = 0$

Asympototes is exist ~~o~~ for X and Y axis.

5 Intersection with axies.

① For X axis : In eq^n of curve putting $y = 0$

$\therefore x(x-3a)^2 = 0$

$\therefore x = 3a$

② For Y axis : In eq^n of curve putting $X = 0$

$9ay^2 = 0$

$Y = 0$

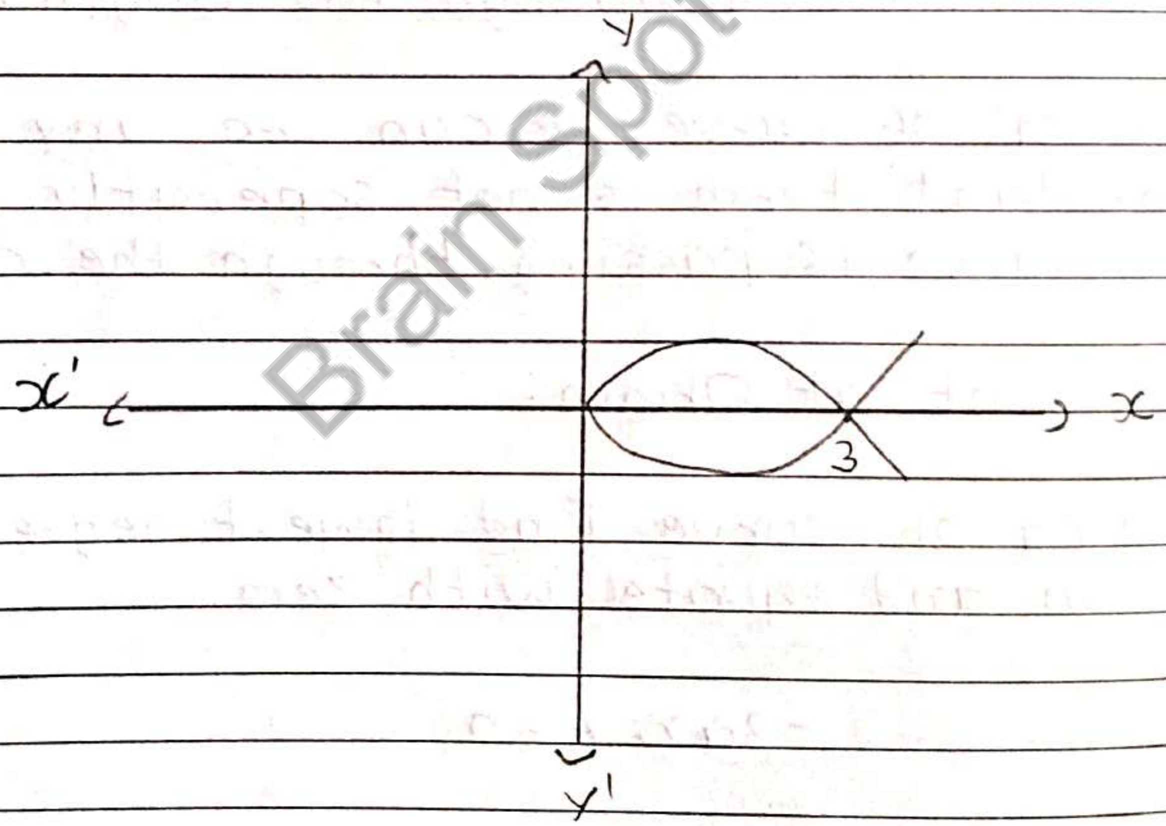
6 Region :

$$y^2 = \frac{x(x-3a)^2}{9a}$$

$$\therefore y = \frac{(x-3a)}{3}\sqrt{\frac{x}{a}}$$

$$0 \leq x \leq 3a$$

Y become imegnary For $0 \leq x < 3a$.



6 $x^3 + y^3 - 3axy = 0$

Given eq^n of curve $x^3 + y^3 - 3axy = 0$ — (1)

1 Symmetry: In the given eq^n of curve, if we interchange x and y eq^n of curve remain unchange.

So, curve is symmetry above $x = y$ line.

2 Curve passing through the Origin:

In eq^n of curve we can see any constant turm is not seperatly.
So, Curve is passing through the origin.

3 Tangent at Origin:

In eq of curve Find lowest degree turm and equate with zero.

$$\therefore -3axy = 0$$

$$\therefore x = 0, \quad y = 0$$

At Origin we get 2 different tangent.
So, Origin is become a node point.

4 Intersection with axis.

Intersection For $Y = x$ line,
take $x = Y$ in eq^n 1

$$\therefore x^3 + x^3 = 3ax^2$$

$$\therefore x = \frac{3a}{2}, \quad Y = \frac{3a}{2}$$

So, Curve into the $Y = x$ line at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

5 Region.

If we take $x = (-x)$ and $Y = (-Y)$
So, Curve does not go in 3^{rd} quadrant.

6 Asymptotes:

(i) Parallel to x axis = NO

(ii) Parallel to Y axis = NO

(iii) Oblique Asymptotes : $Y = mx + c$

Now, For Asymptotes, we take

$$\phi_n(m) = \phi_3(m) = 1 + m^3$$

$\therefore \phi_{n-1}(m) = \phi_2(m) = -3am$

Now, we take $\phi_n(m) = 0$

$\therefore 1 + m^3 = 0$

$\therefore (m+1)(m^2 - m + 1) = 0$

$m^2 - m + 1 = 0$ has imaginary root.

So, $m + 1 = 0$, $m = -1$

Now C Find as Follow

$$C = -\frac{\phi_{(n-1)}(m)}{\phi_n'(m)} = \frac{-(-3am)}{3m^2}$$

$$\therefore C = \frac{a}{m}$$

$$\therefore C = -a$$

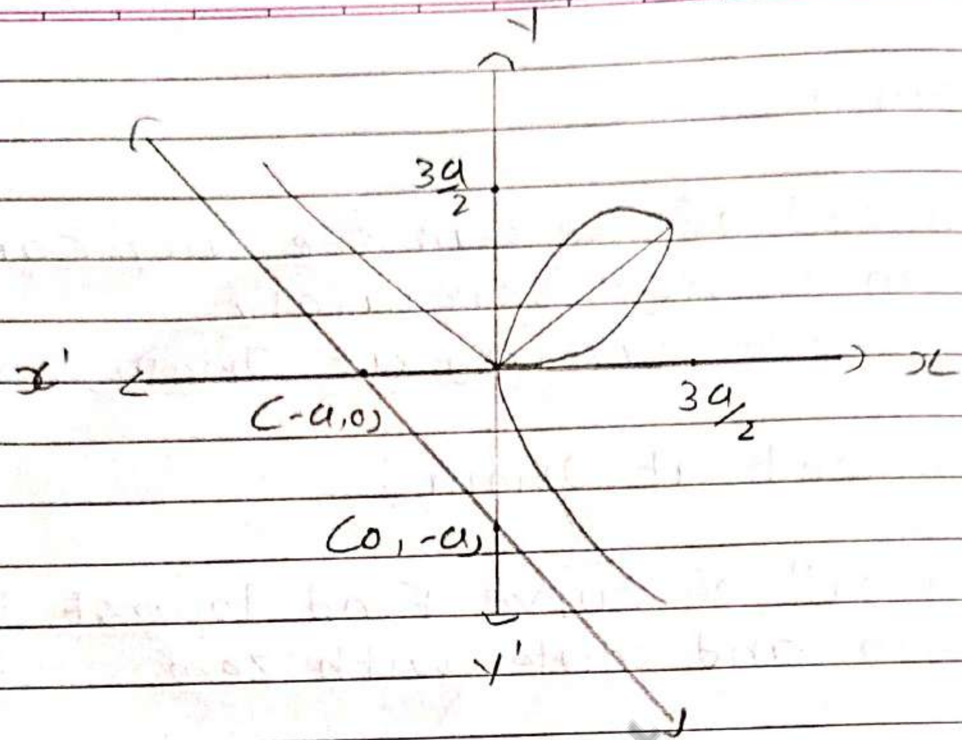
Now oblique Asymptotes

$\therefore y = mx + c$

$\therefore y = -m - a$

$\therefore x + y = -a$

$\therefore (0, -a), (-a, 0)$



7 $y^2(a^2+x^2) = x^2(a^2-x^2)$

Given eq^n of curve $y^2a^2+y^2x^2 = x^2a^2-x^4$

1 Symmetry:

- In eq^n of curve remains unchange when replasing x by $-x$. So the curve is Symmetry above Y axis.

- In eq^n of curve remains unchange when replasing x by $-x$ and y by $-y$. So the curve has symmetry above opposite quadrant.

- In eq^n of curve remains unchange when replasing y by $-y$. So the curve has symmetry above X axis.

2 Origin:

In eq^n of we can see any constant term is not seperately. 1)
So, Curve passing at Origin.

3 Tangent at Origin:

In eq^n of curve Find lowest degree term and equate with zero.

lowest degree term is $y^2a^2 = 0$
$x^2a^2 = 0$

$\therefore x = 0, \; y = 0$

two tangent are different So, Origin become a node.

4 Intersection with axis:

① For X axis: In eq^n of curve putting
$y = 0$

$\therefore x^2a^2 = x^4$

$\therefore x = a$

② For Y axis: In eq^n of curve putting
$X = 0$

$\therefore\ y^2 a^2 = 0$

$\therefore\ y = 0$

5 Region :

$$y^2 = \frac{x^2(a^2-x^2)}{a^2+x^2} \rightarrow y = x\sqrt{\frac{a^2-x^2}{a^2+x^2}}$$

$$x^2 = \frac{y^2(a^2+x^2)}{a^2-x^2} \rightarrow x = y\sqrt{\frac{a^2+x^2}{a^2-x^2}}$$

$$-a \le x \le a$$

6 Asymptotes:

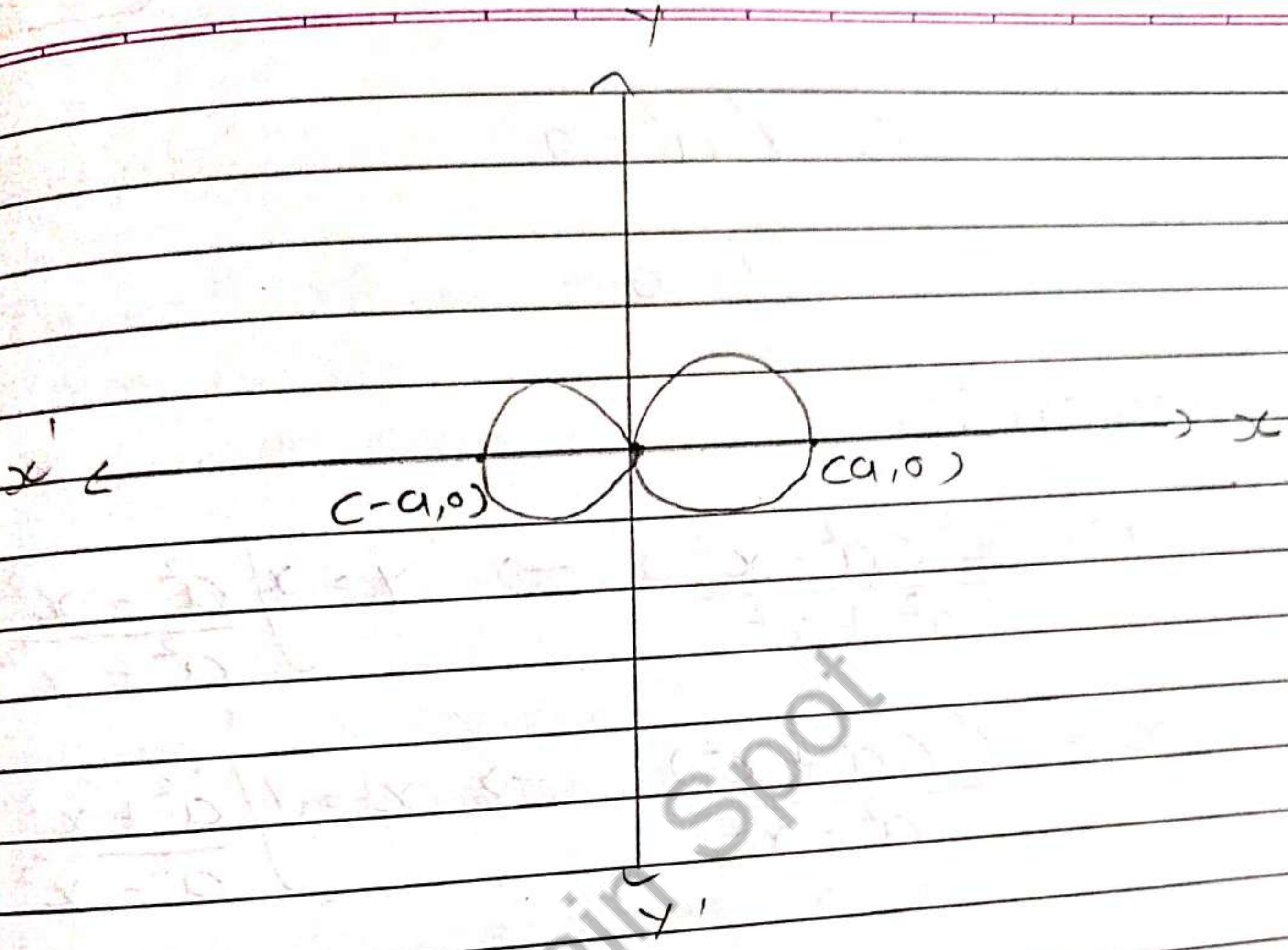
(i) Parallel to x axis : No

(ii) Parallel to y axis : No

(iii) Oblique Asympototes: $y = mx + c$

$\phi_n(m) = \phi_2(m) = m^2 - 1$

$\therefore \phi_{n-1}(m) = \phi_1(m) =$


Y
X
X'
(a,0)
(-a,0)
Y'

Unit-3 Curve Tracing

* Task 2 : Polar curves

1 $r = a(1 - \cos\theta)$

Here given eq^n of the curve is $r = a(1-\cos\theta)$
We want to trace the curve by following steps.

1 Symmetry :

In eq^n of curve remains unchange by ~~e~~ replasing θ by $-\theta$. that's why curve has inital line symmetry.

2 Passing through the Pole :

When we take $r=0$ and eq^n give such θ then curv passing through the pole.

$$\therefore r = a(1 - \cos\theta)$$

$$\therefore a(1 - \cos\theta) = 0$$

$$\therefore 1 - \cos\theta = 0$$

$$\therefore \cos\theta = 1$$

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$\therefore \theta = 0$

Here, we have such value of θ that's why curve passing through the pole.

3 Tangent at Pole :

We take $r=0$ and we get $\theta=0$ then curve has tangent at Pole.

4 Diration of tangent:

Diration of tangent we find by formula,

$$\tan\phi = \frac{r}{dr/d\theta}$$

$$= \frac{a(1-\cos\theta)}{a\sin\theta}$$

$$= \frac{1-\cos\theta}{\sin\theta}$$

$$= \frac{2\sin^2\theta/2}{2\sin\theta/2 \cdot \cos\theta/2}$$

$$\tan\phi = \tan\frac{\theta}{2}$$

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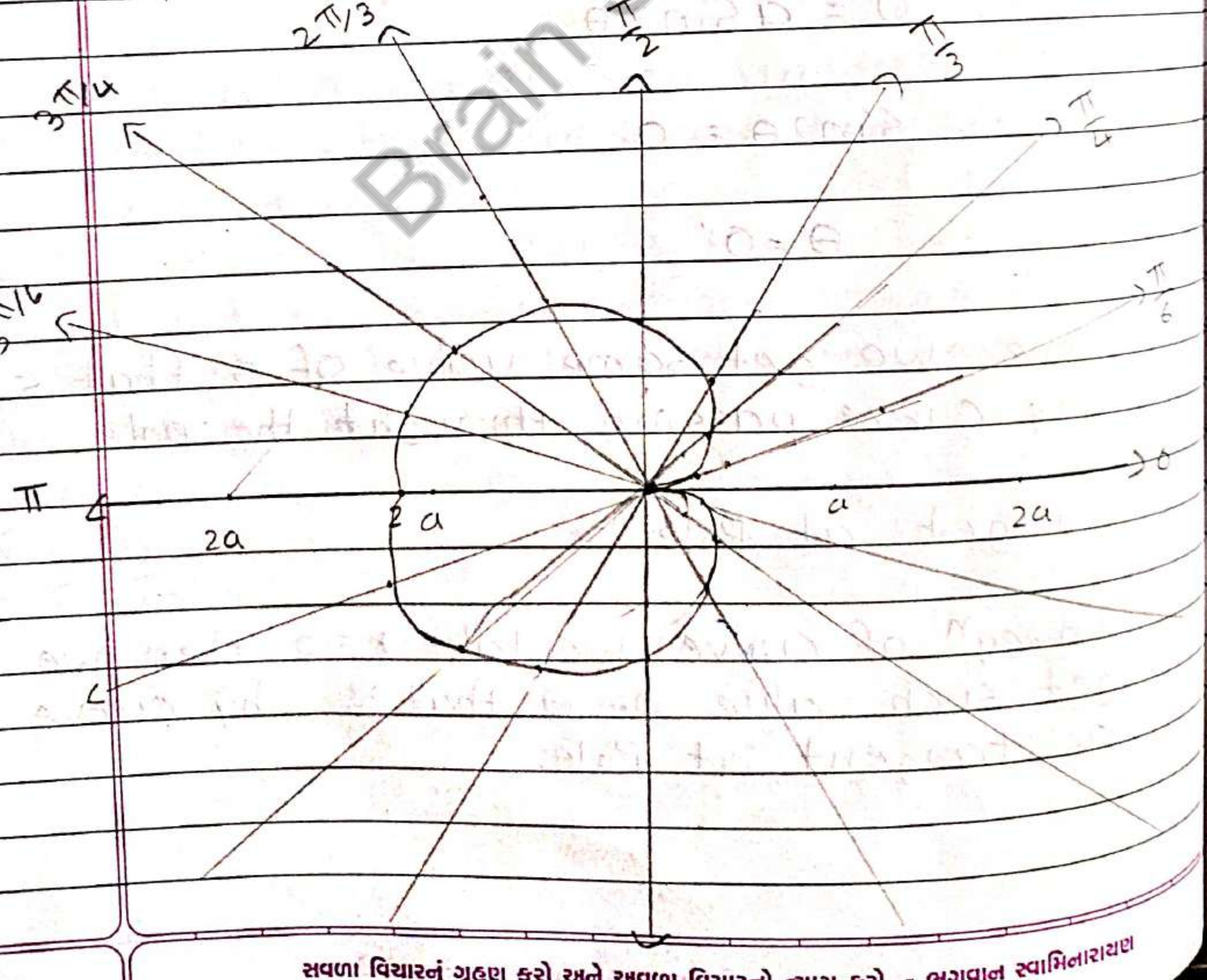
$$\therefore \phi = \frac{\theta}{2}$$

We take $\theta = 0^\circ$ then $\phi = 0$, So radius and tangent vector are equal

We take $\theta = \pi$ then $\phi = \pi/2$, So radius and tangent vector are perpendicular.

5 table.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	0	0.3a	0.29a	0.5a	a	1.5a	1.7a	1.86a	2a



2 $r = a\sin 2\theta$

1 Symmetry :

In eq^n of Curve we change θ by $-\theta$ then eq^n of Curve remain unchange that's why curve as symmetry above inital line.

2 Passing through the pole :-

In eq^n of Curve we take $r = 0$ then we get such θ value.

$$\therefore \quad 0 = a\sin 2\theta$$

$$\therefore \quad \sin 2\theta = 0$$

$$\therefore \quad \theta = 0^\circ$$

Here, we get same value of θ that's why curve passing through the pole.

3 Tangent at Pole :

In eq^n of curve we take $r = 0$ then we get such value of θ that's why curve as tangent at Pole.

4 Diration of tangent

Diration of tangent we find by this formula.

$$\tan\phi = \frac{r}{dr/d\theta}$$

$$= \frac{a\sin 2\theta}{2a\cos 2\theta}$$

$$\tan\phi = \frac{1}{2}\tan 2\theta$$

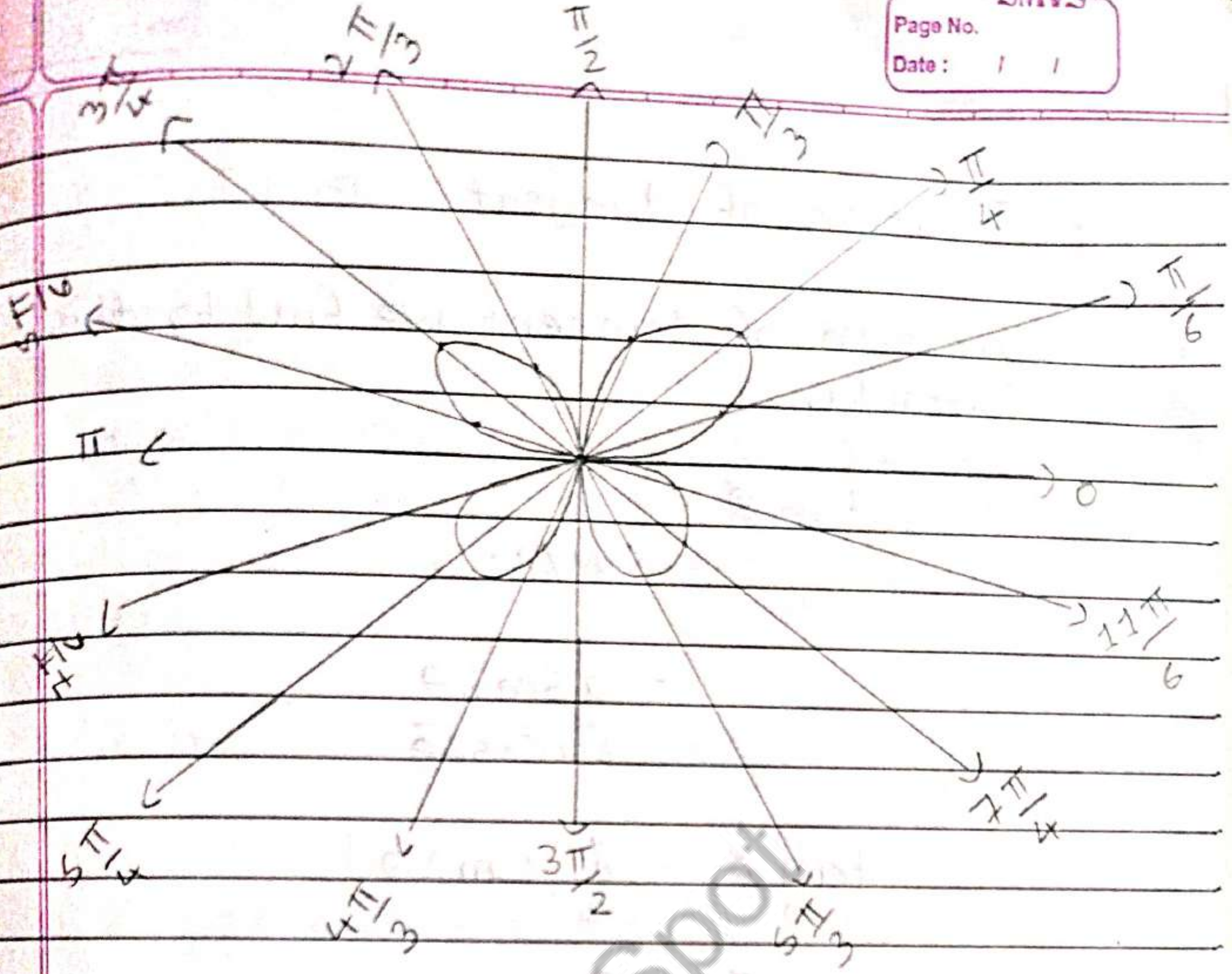
$$\phi = \theta$$

We take $\theta = 0$ then we get $\phi = 0$ that's why radius and tangent vector are equal.

We take $\theta = \pi/2$ then we get $\phi = \pi/2$ that's why radius and tangent vector are perpendicular.

5 Table:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	0	$0.86a$	$1a$	$0.86a$	0	$-0.86a$	$-1a$	$-0.86a$	0



3 $r^2 = a^2 \cos 2\theta$

1 Symmetry: In given eq^n of we replasing θ by $-\theta$ then eq^n of curve remain unchange then curve has symmetry above initial line.

If eq^n of curve we replasing r by $-r$ and θ by $-\theta$ then eq^n of curve remain unchange that's why curve has symmetry above $\pi/2$ line.

2 Passing through the pole:

In eq^n of Curve we take $r = 0$ then we get such value of θ

$$\therefore r^2 = a^2 \cos 2\theta$$

$$\therefore a^2 \cos 2\theta = 0$$

$$\therefore \cos 2\theta = 0$$

$$\therefore \theta = \frac{\pi}{2}$$

We get such value of θ that's why curve passing through the pole.

3 Tangent at Pole:

~~W~~ We take $r = 0$ then we get such value of θ that's why curve has tangent at Pole.

4 Diration of tangent:

Find Diration of tangent use by this formula.

$$r = a\sqrt{\cos 2\theta} \quad , \quad \frac{dr}{d\theta} = \frac{-a \cdot 2\sqrt{\cos 2\theta} \cdot \sqrt{\sin 2\theta}}{2\sqrt{\cos 2\theta}}$$

$$= -a\sqrt{\sin 2\theta}$$

$$\therefore \tan\phi = \frac{r}{dr/d\theta} = \frac{a\sqrt{\cos 2\theta}}{-a\sqrt{\sin 2\theta}}$$

$\therefore \tan\phi = -\cot 2\theta$

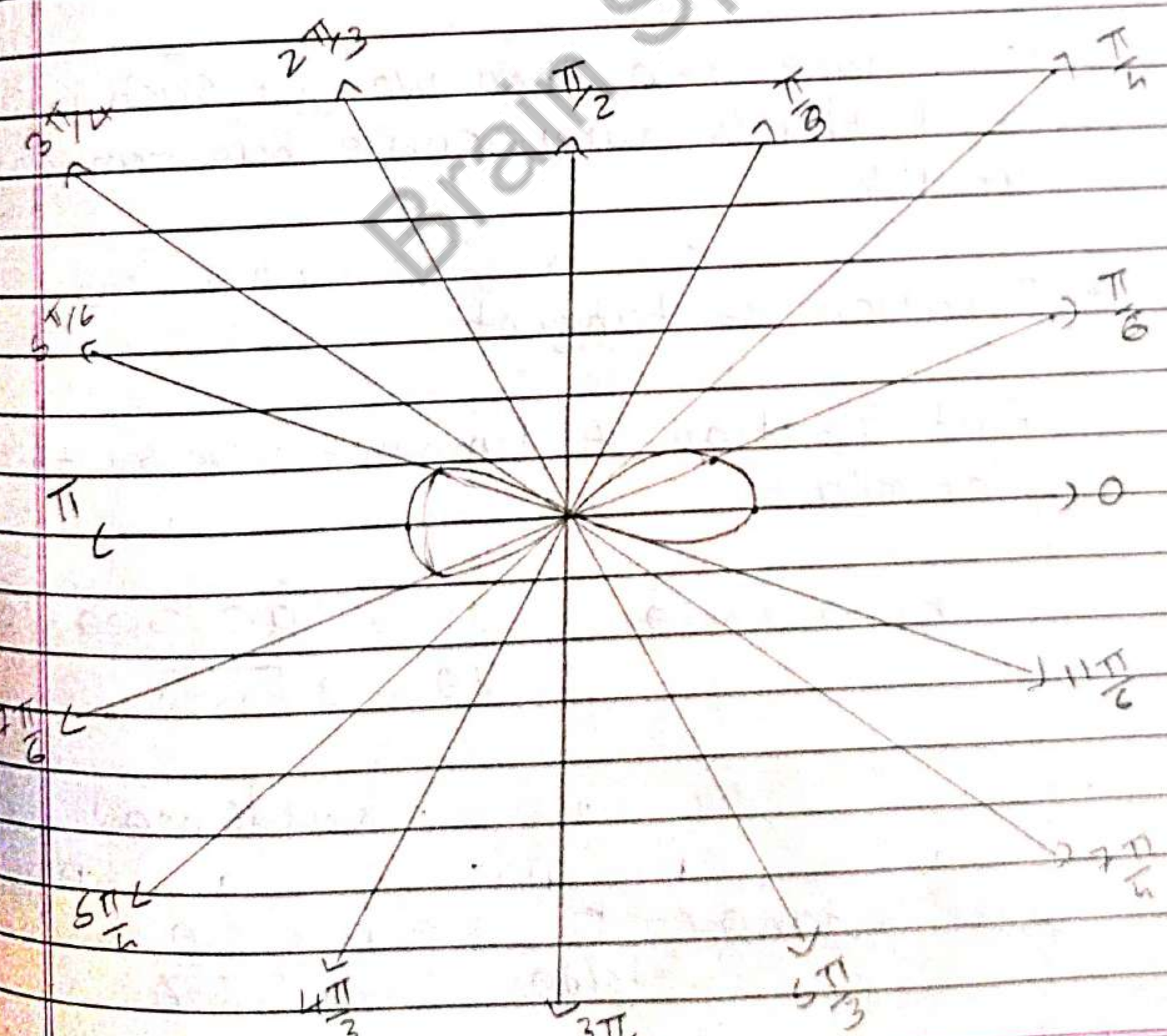
5 Region: Here given eq^n curve $r^2 = a^2\cos 2\theta$ and we know that $\cos\theta$ +ve in 1^{st} and 4^{th} qurdrant.

Now, $-\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$

$\therefore -\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$

6 table:

θ	0	$\pi/6$	$\pi/4$	$3\pi/4$	$\frac{5\pi}{6}$	π
r	a	0.7a	0	0	0.7a	a



4 $r = a\sin 3\theta$

1 Symmetry :

In eq^n of curve we replasing θ by $-\theta$ then eq^n of curve remain unchange that's why curve has symmetry above initial line.

In eq^n of Curve we replasing θ by $\pi - \theta$ then eq^n of curve remain unchange that's why curve has symmetry above $\pi/2$ line.

2 Passing through the Pole

We take $r=0$ then we get such value of θ

$$\therefore r = a\sin 3\theta$$

$$\therefore a\sin 3\theta = 0$$

$$\therefore \theta = 0$$

3 ~~To~~ Here, we get such value of θ then curve has passing through the pole.

3 Tangent at Pole:

We take $r=0$ then we get such value of $\theta=0$ that's why curve as tangent at Pole.

4 Diraction of tangent:

We Find Diraction of tangent by use this Formula.

$$\tan\phi = \frac{r}{dr/d\theta}$$

$$= \frac{a\sin 3\theta}{3a\cos 3\theta}$$

$$\therefore \tan\phi = \frac{1}{3}\tan 3\theta$$

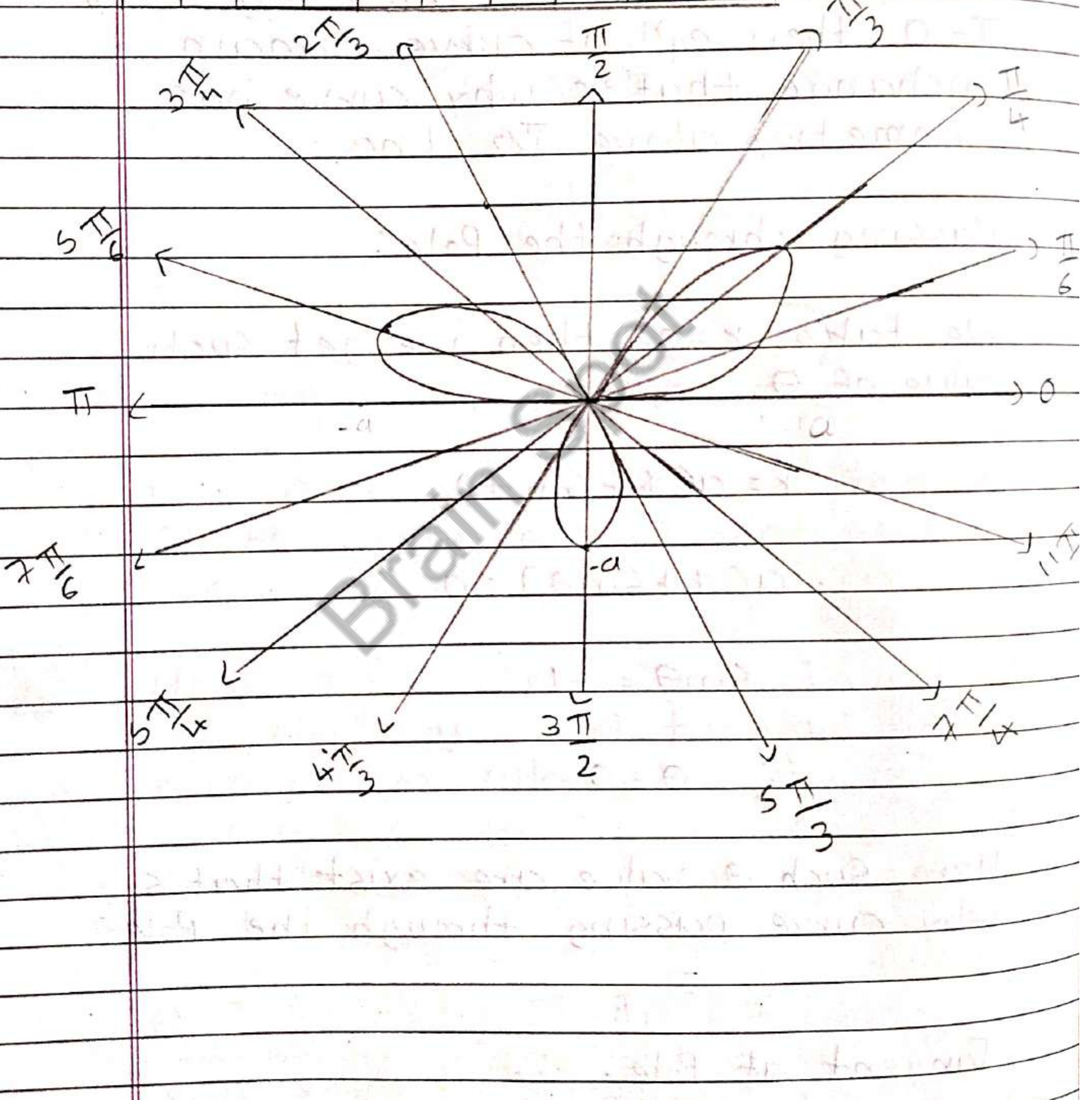
$$\therefore \phi = \theta$$

We take $\phi=0$ then $\theta=0$. So, radius and tangent vector are ~~prependicular~~ equal.

We take $\phi=\pi/2$ then $\theta=\pi/2$. So, radius and tangent vector are prependicular.

5 Table:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	0	a	$0.7a$	0	$-a$	$-a$	$0.7a$	a	0



5 $r = a(1 + \sin\theta)$

1 Symmetry:

In eq^n of curve we replasing θ by $\pi - \theta$ then eq^n of curve remain unchange that's why curve has symmetry above $\pi/2$ line.

2 Passing through the Pole:

We take $r = 0$ then we get such value of θ

$$\therefore r = a(1 + \sin\theta)$$

$$\therefore a(1 + \sin\theta) = 0$$

$$\therefore \sin\theta = -1$$

$$\therefore \theta = 0'$$

Here, such θ value are exist that's why curve passing through the Pole.

3 Tangent at Pole:

We take $r = 0$ and we get such value of $\theta = 0$ so, curve has tangent at Pole.

4 Diration of tanget

Diration of tanget we Find by this formula,

$$\therefore \tan\phi = \frac{r}{dr/d\theta}$$

$$= \frac{a(1+\sin\theta)}{a\cos\theta}$$

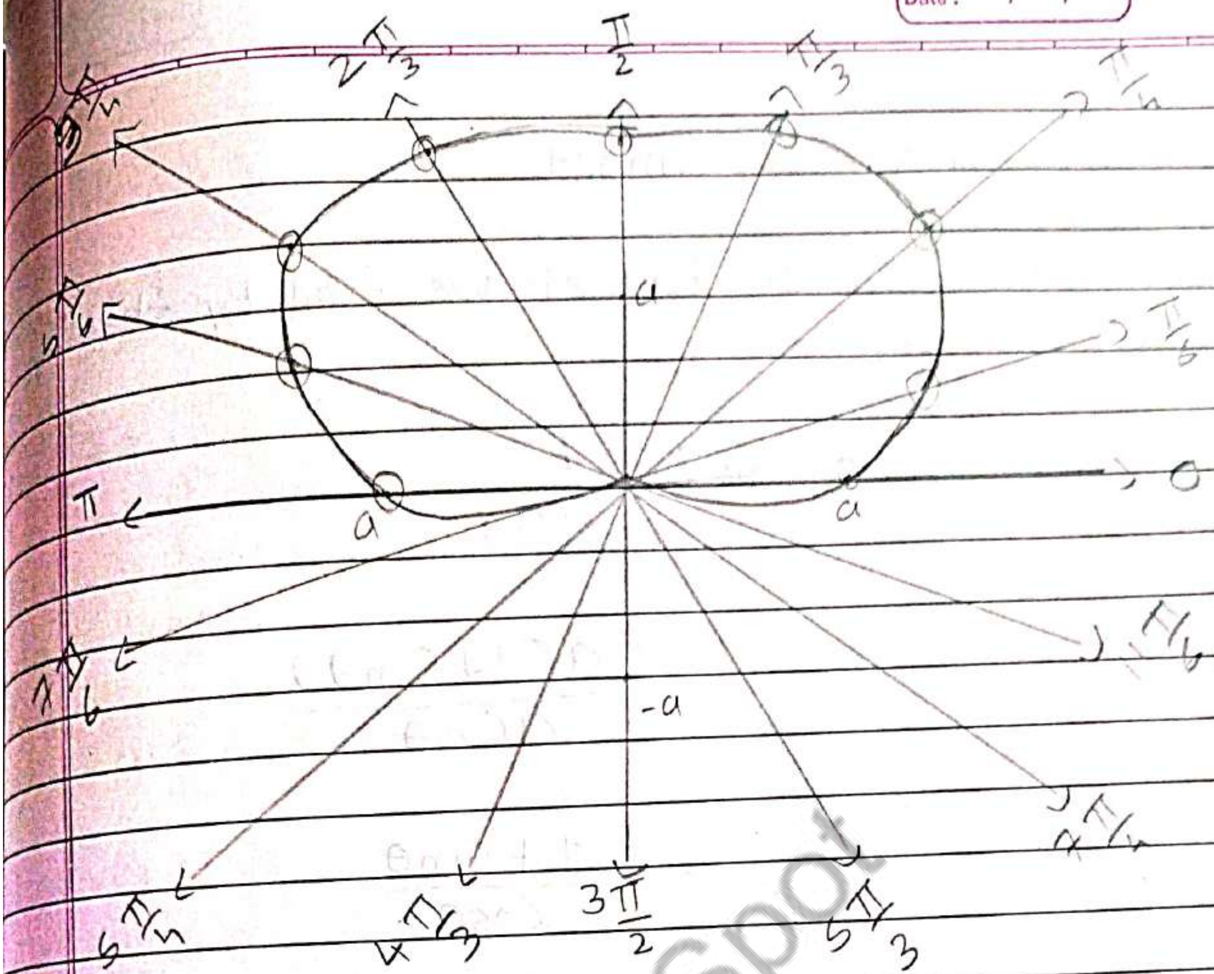
$$= \frac{1+\sin\theta}{\cos\theta}$$

Here, $\theta \to 0$, then $\tan\phi = 0$, then $\phi = 0$
So, the radius and tangent vector are equal.

Here, $\theta \to 0$ then $\phi = \pi/2$.
So, the radius and tangent vector are prependicular.

5 Table :

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	a	1.5 a	1.7 a	1.86 a	2a	1.86 a	1.7 a	1.5 a	a



6 $r = a\cos 2\theta$

1 Symmetry:

In given eq^n of curve we replasing θ by $-\theta$ eq^n of curve remain unchange then curve has symmetry above initial line.

2 Passing through the Pole:

given eq^n of curve we take $r = 0$ then we get such value of θ

$$\therefore r = a\cos 2\theta$$

$$\therefore a\cos 2\theta = 0$$

$$\therefore 2\theta = \pi$$

$$\therefore \theta = \frac{\pi}{2}$$

Here, We get $\theta = \pi/2$ then eq^n of curve passing through the Pole.

3 Tangent at Pole:

We take $r=0$ then we get $\theta = \pi/2$. So, curve has tangent at Pole.

4 Diraction of tangent

Diraction of e tangent we Find use of this formula,

$$\tan\phi = \frac{r}{\partial r/\partial\theta}$$

$$= \frac{a\cos 2\theta}{-2a\sin 2\theta}$$

$$= -\frac{1}{2}\cot 2\theta$$

Here, we take $\theta = \pi/2$ then $\phi = 0$.
So, radius and tangent vector are equal.

We take $\theta = 0$ then $\phi = \pi/2$
so, radius and tangent vector are prependicular.

Table

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
r	$1a$	$0.5a$	0	$-0.5a$	$-0.6a$	$-0.8a$	$-0.9a$	$-1a$	$1a$