

Unit-1 Differential Calculus

* Task-1 Limit, Continuity and Differentiability

1 Evaluate $\lim_{x \rightarrow -1} \frac{(x+3)(2x-1)}{x^2+3x-2}$

$$= \lim_{x \rightarrow -1} \frac{(x+3)(2x-1)}{x^2+3x-2}$$

$$= \frac{(-1+3)(2(-1)-1)}{(-1)^2+3(-1)-2}$$

$$= \frac{(2)(-2-1)}{1-3-2}$$

$$= \frac{2(-3)}{-4}$$

$$= \frac{3}{2}$$

2 Evaluate $\lim_{x \rightarrow -1} \frac{2x^3-3x^2-3x+2}{3x^3+2x^2-11x-10}$

$$= \lim_{x \rightarrow -1} \frac{(x+1)(2x^2-5x+2)}{(x+1)(3x^2-x-10)}$$

$$= \lim_{x \rightarrow -1} \frac{2x^2 - 5x + 2}{3x^2 - x - 10}$$

$$= \frac{2(-1)^2 - 5(-1) + 2}{3(-1)^2 - (-1) - 10}$$

$$= \frac{2 + 5 + 2}{3 + 1 - 10}$$

$$= \frac{9}{-6}$$

$$= -\frac{3}{2}$$

$$= -\frac{3}{2}$$

3 Evaluate $\lim_{x \rightarrow 1/5} \frac{125x^3 - 1}{625x^4 - 1}$

$$= \lim_{x \rightarrow 1/5} \frac{(5x)^3 - (1)^3}{(5x)^4 - (1)^4}$$

$$= \lim_{x \rightarrow 1/5} \frac{(5x)^3 - (1)^3}{x-1} \times \frac{x-1}{(5x)^4 - (1)^4}$$

$$= \lim_{x \rightarrow 1} \frac{3x^3 - 1}{4x^4 - 1}$$

$$= \frac{3(1)^{3-1}}{4(1)^{4-1}}$$

$$= \frac{3}{4}$$

4 Discuss the Continuity of

$$f(x) = \frac{5}{x-1}, \quad x \neq 1.$$

$$= 1, \quad x = 1$$

$$\therefore \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{5}{x-1}$$

$$= \lim_{x \rightarrow 1} \frac{5}{1-1}$$

$$= \infty$$

$$\therefore \lim_{x \rightarrow 1} f(x) \neq f(1)$$

$\therefore f(x)$ is not Continuity at $x = 1$

5 Discuss the continuity of

$$f(x) = 2x + 3, \quad x < 1$$

$$5, \quad x = 1$$

$$2 + 3x, \quad x > 1$$

$$\therefore \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x)$$

$$\therefore \lim_{x \rightarrow 1^+} 2 + 3x = \lim_{x \rightarrow 1^-} 2x + 3$$

$$\therefore 2 + 3(1) = 2(1) + 3$$

$$\therefore 5 = 5$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^-} f(x) = f(1)$$

$f(x)$ is continuity at $x = 1$

6 Let $f(x) = \sqrt{2x-1}$ than find $f'(5)$

$$\therefore f'(5) = \lim_{h \rightarrow 0} \frac{f(x_0+h) - f(x_0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{10+2h}-1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{9+2h}-3}{h} \times \frac{\sqrt{9+2h}+3}{\sqrt{9+2h}+3}$$

$$= \lim_{h \rightarrow 0} \frac{9+2h+9}{h(\sqrt{9+2h}+3)}$$

$$= \lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{9+2h}+3)}$$

$$= \frac{2}{\sqrt{9+2(0)}+3}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

7 If $f(x) = \sin^2 x$ then find $f'(x)$ by definition.

$$f'(x) = \lim_{h \rightarrow 0} \frac{(\sin(x+h))^2 - (\sin x)^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(\sin x \cosh + \cos x \sinh)^2 - \sin^2 x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sin^2 x \cdot \cosh (\cosh + 1) + \dots}{h}$$

$$\begin{aligned}
 & 2 \sin x \cos x \cosh \cos x - \frac{\sinh}{h} + \\
 & \cos^2 x \cdot \frac{\sinh}{h} \cdot \sinh \\
 = & \lim_{h \rightarrow 0} \frac{\sin^2(\cos) (\cos + 1) + 2 \sin(\cos)}{\cos x (1) + \cos x \sin 0} \\
 = & 2 \sin x \cos x
 \end{aligned}$$

8 Evaluate the limit $\lim_{x \rightarrow -8} \frac{\sqrt{1-x} - 3}{2 + 3\sqrt{x}}$

$$= \lim_{x \rightarrow -8} \frac{(1-x)^{\frac{1}{2}} - 3}{2 + (x)^{\frac{1}{3}}}$$

$$= \lim_{x \rightarrow -8} \frac{\frac{1}{2} (1-x)^{-\frac{1}{2}+1}}{\frac{1}{3} (x)^{-\frac{2}{3}}}$$

$$= \lim_{x \rightarrow -8} \frac{\frac{1}{2} \sqrt{1-x}}{\frac{1}{3} (x)^{-\frac{2}{3}}}$$

$$= \frac{1}{2} \frac{\sqrt{1-(-8)} \times 3}{(2^3)^{-\frac{2}{3}}}$$

$$= 2$$

* Task 2 : Sandwich Theorem

1 Find $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$

We know that,

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

By Sandwich Theorem,

$$\therefore -1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

$$\therefore -x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$$

$$\therefore \lim_{x \rightarrow 0} -x^2 \leq \lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) \leq \lim_{x \rightarrow 0} x^2$$

$$\therefore 0 \leq \lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) \leq 0$$

By Sandwich Theorem $x^2 \sin\left(\frac{1}{x}\right) = 0$

2 IF $5x \leq f(x) \leq 2x^2 + 2$, $\forall x \in \mathbb{R}$ then
Find $\lim_{x \rightarrow 2} f(x)$

Given, $5x \leq f(x) \leq 2x^2 + 2$

$$\therefore \lim_{x \rightarrow 2} 5x \leq \lim_{x \rightarrow 2} f(x) \leq \lim_{x \rightarrow 2} 2x^2 + 2$$

$$\therefore 5(2) \leq \lim_{x \rightarrow 2} f(x) \leq 2(2)^2 + 2$$

$$\therefore 10 \leq \lim_{x \rightarrow 2} f(x) \leq 10$$

By Sandwich Theorem, $\lim_{x \rightarrow 2} f(x) = 10$.

3 Find $\lim_{x \rightarrow 0} g(x)$, if $3 - x^3 \leq g(x) \leq 3 \sec x$

for all $x \in \mathbb{R}$.

Given, $3 - x^3 \leq g(x) \leq 3 \sec x$

$$\therefore \lim_{x \rightarrow 0} (3 - x^3) \leq \lim_{x \rightarrow 0} g(x) \leq \lim_{x \rightarrow 0} (3 \sec x)$$

$$\therefore 3 \leq \lim_{x \rightarrow 0} g(x) \leq 3$$

By Sandwich Theorem, $\lim_{x \rightarrow 0} g(x) = 3$

4 Evaluate $\lim_{x \rightarrow \infty} \frac{\cos^2(2x)}{3-2x}$

We know that, $0 \leq \cos^2 2x \leq 1$

By Sandwich theorem,

$$\therefore 0 \leq \cos^2 2x \leq 1$$

$$\therefore 0 \leq \frac{\cos^2 2x}{3-2x} \leq \frac{1}{3-2x}$$

$$\therefore \lim_{x \rightarrow \infty} 0 \leq \lim_{x \rightarrow \infty} \frac{\cos^2(2x)}{3-2x} \leq \lim_{x \rightarrow \infty} \frac{1}{3-2x}$$

$$\therefore 0 \leq \lim_{x \rightarrow \infty} \frac{\cos^2(2x)}{3-2x} \leq 0$$

By Sandwich Theorem, $\lim_{x \rightarrow \infty} \frac{\cos^2(2x)}{3-2x} = 0$

5 Prove that $\lim_{x \rightarrow \infty} \sqrt{x} \left(1 + \sin^2\left(\frac{2\pi}{x}\right) \right) = \infty$

We know that, $0 \leq \sin^2\left(\frac{2\pi}{x}\right) \leq 1$

By Sandwich theorem,

$$\therefore 0 \leq \sin^2\left(\frac{2\pi}{x}\right) \leq 1$$

$$\therefore 1 \leq 1 + \sin^2\left(\frac{2\pi}{x}\right) \leq 2$$

$$\therefore \sqrt{x} \leq \sqrt{x} \left(1 + \sin^2\left(\frac{2\pi}{x}\right)\right) \leq 2\sqrt{x}$$

$$\therefore \lim_{x \rightarrow 0} \sqrt{x} \leq \lim_{x \rightarrow 0} \sqrt{x} \left(1 + \sin^2\left(\frac{2\pi}{x}\right)\right) \leq \lim_{x \rightarrow 0} 2\sqrt{x}$$

$$\therefore 0 \leq \lim_{x \rightarrow 0} \sqrt{x} \left(1 + \sin^2\left(\frac{2\pi}{x}\right)\right) \leq 0$$

By Sandwich theorem,

$$\lim_{x \rightarrow 0} \sqrt{x} \left(1 + \sin^2\left(\frac{2\pi}{x}\right)\right) = 0$$

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* Task 3 : Indeterminate Forms

$$1 \quad \lim_{x \rightarrow 1} \frac{x^x - x}{x - 1 - \log x}$$

Given $l = \lim_{x \rightarrow 1} \frac{x^x - x}{x - 1 - \log x}$ $\left(\frac{0}{0}\right)$ form

By L' Hospital,

$$\therefore l = \lim_{x \rightarrow 1} \frac{x^x(1 + \log x) - 1}{1 - \frac{1}{x}} \quad \left(\frac{0}{0}\right) \text{ form}$$

By L' Hospital,

$$\therefore l = \lim_{x \rightarrow 1} \frac{x^x \left(\frac{1}{x}\right) + (1 + \log x)(1 + \log x)}{+ 1/x^2}$$

$$\therefore l = 1 + 1$$

$$= 2$$

$$2. \quad \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$$

given, $l = \lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$ $\left(\frac{0}{0}\right)$ form

By L' Hospital,

$$l = \lim_{x \rightarrow 0} \frac{1 - \cos x}{3x^2} \quad \left(\frac{0}{0} \right) \text{ form,}$$

By L' Hospital,

$$\therefore l = \lim_{x \rightarrow 0} \frac{\sin x}{6x}$$

$$\therefore l = \lim_{x \rightarrow 0} \frac{1}{6} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$l = \frac{1}{6}$$

3 $\lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}$

$$= \lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} - \lim_{x \rightarrow 0} e$$

$$\frac{\lim_{x \rightarrow 0} x}{\lim_{x \rightarrow 0} x}$$

$$= \lim_{x \rightarrow 0} \frac{e \log(1+x)^{\frac{1}{x}} - e}{x}$$

$$= \lim_{x \rightarrow 0} e^{\frac{1}{x} \log(1+x)} \left[-\frac{1}{x^2} \log(1+x) + \frac{1}{x} \cdot \frac{1}{1+x} \right]$$

$$= e \lim_{x \rightarrow 0} \frac{-\log(1+x)(1+x)}{x^2(1+x)}$$

$$= e \lim_{x \rightarrow 0} \frac{-1 - \log(1+x) + 1}{2x + 3x^2}$$

$$= e \lim_{x \rightarrow 0} \frac{-\frac{1}{1+x}}{2+6x} = \frac{-e}{2}$$

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4 Determine value of a, b, c so that

$$\lim_{x \rightarrow 0} \frac{ae^{-x} - b\cos x + ce^{-x}}{x\sin x} = 2$$

given $l = \lim_{x \rightarrow 0} \frac{ae^{-x} - b\cos x + ce^{-x}}{x\sin x} = 2$

$$\therefore \lim_{x \rightarrow 0} \frac{ae^{-x} - b\cos x + ce^{-x}}{x\sin x} = 2 \quad \left(\frac{0}{0}\right)$$

By L'Hospital,

$$\therefore \lim_{x \rightarrow 0} \frac{-ae^{-x} + b\sin x - ce^{-x}}{x\cos x + \sin x} = 2$$

$$\therefore \frac{-ae^{-0} + b\sin 0 - ce^{-0}}{0 + 0} = 2$$

$$\therefore \cancel{b\sin 0} = 2$$

$$\therefore \boxed{b = 2}$$

$$\therefore -a - c = 0 \rightarrow a = -c \quad \text{--- (1)}$$

$$l = \lim_{x \rightarrow 0} \frac{ae^x - b\cos x + ce^{-x}}{x \sin x} = \textcircled{2}$$

$$\therefore a - b + c = 0 \quad - \textcircled{2}$$

eqⁿ 1 and 2 according,
 $\therefore c - b + c = 0$

$$\therefore 2c = b \quad \& \quad 2a = b$$

$$\rightarrow l = \lim_{x \rightarrow 0} \frac{ae^x + b\sin x - ce^{-x}}{x \cos x + \sin x} = 0$$

Applying L' Hospital rule,

$$\therefore \lim_{x \rightarrow 0} \frac{ae^x + b\cos x + ce^{-x}}{\cos x + x \sin x + \cos x} = 0$$

$$\therefore \frac{a + b + c}{2} = 0$$

$$\therefore 2a + 2a = 4$$

$$\therefore \boxed{a = 1}$$

$$\therefore 2a = b \quad \therefore \boxed{b = 2}$$

$$\therefore 2c = b \quad \therefore \boxed{c = 1}$$

$$7 \quad \lim_{x \rightarrow \frac{\pi}{2}} \frac{\log\left(x - \frac{\pi}{2}\right)}{\tan x}$$

By L' Hospital,

$$l = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1/x - \pi/2}{\sec^2 x}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos^2 x \cdot \frac{1}{x - \pi/2}}{x - \pi/2}$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}} -2 \cos \frac{\pi}{2} \cdot \sin \frac{\pi}{2}$$

$$= 0$$

$$8 \quad \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{1}{3x}}$$

$$l = \lim_{x \rightarrow 0} \left[\frac{a^x + b^x + c^x}{3} \right]^{\frac{1}{3x}}$$

$$\therefore \log l = \lim_{x \rightarrow 0} \left[\frac{a^x + b^x + c^x}{3x \times 3} \right] \left(\frac{0}{0} \right)$$

$$= \lim_{x \rightarrow 0} \frac{3}{a^x + b^x + c^x} \cdot \frac{a^x \log a + b^x \log b + c^x \log c}{3}$$

$$\therefore \log l = \frac{\log a + \log b + \log c}{g}$$

$$\therefore l = \frac{abc}{g}$$

$$9 \quad \lim_{x \rightarrow 0} [\cos x]^{\cot x}$$

$$l = \lim_{x \rightarrow 0} (\cot x \cdot \log(\cos x))$$

$$l = \lim_{x \rightarrow 0} \cot x \cdot \log(\cos x)$$

$$= \lim_{x \rightarrow 0} \frac{\cot x}{1/\log(\cos x)} \quad \left(\frac{\infty}{\infty} \right)$$

$$= \lim_{x \rightarrow 0} \frac{-\operatorname{cosec}^2 x}{-\frac{\cos x}{\sin x}}$$

$$= \lim_{x \rightarrow 0} \tan x$$

$$= 1$$

$$5 \quad \lim_{x \rightarrow y} \left[\frac{x^y - y^x}{x^x - y^y} \right]$$

$$e = \lim_{x \rightarrow y} \frac{y x^{y-1} - y^x \log y}{x^x (1 + \log x) - 0}$$

$$= \lim_{x \rightarrow y} \frac{y \cdot y^{y-1} - y^y \log y}{y^y (1 + \log y)}$$

$$= \frac{y^y (1 - \log y)}{y^y (1 + \log y)}$$

$$6 \quad \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$$

$$= \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2} \quad \left(\frac{0}{0} \right) \text{ form}$$

Applying L' Hospital rule,

$$= \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{2 \sin x \cdot \cos x - 2x^2}$$

$$= \lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{\sin 2x - 2x^2} \quad \left(\frac{0}{0} \right) \text{ form}$$

Applying L' Hospital rule,

$$= \lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2}{2\cos 2x - 2} \quad \left(\frac{0}{0} \right) \text{ Form}$$

Applying L' Hospital,

$$= \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{-4\sin 2x} \quad \left(\frac{0}{0} \right) \text{ Form}$$

Applying L' Hospital,

$$= \lim_{x \rightarrow 0} \frac{e^x + e^{-x}}{-8\cos 2x}$$

$$= \frac{2}{-8}$$

$$= -\frac{1}{4}$$

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* Task 4: Taylor's and Maclaurin's expansion.

1 Expand $\log(\cos x)$ about $\pi/3$ using Taylor's expansions.

given $f(x) = \log(\cos x)$

By Taylor's Series

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \dots$$

put $a = \frac{\pi}{3}$

$$f\left(x = \frac{\pi}{3}\right) = f\left(\frac{\pi}{3}\right) + \left(x - \frac{\pi}{3}\right) f'\left(\frac{\pi}{3}\right) + \frac{\left(x - \frac{\pi}{3}\right)^2}{2!} f''\left(\frac{\pi}{3}\right) + \frac{\left(x - \frac{\pi}{3}\right)^3}{3!} f'''\left(\frac{\pi}{3}\right) + \dots$$

$\rightarrow f\left(\frac{\pi}{3}\right) = \log(\cos \frac{\pi}{3}) = \log 2$

$\rightarrow f'(x) = \frac{1}{\cos x} (-\sin x) = -\tan x$

$f'\left(\frac{\pi}{3}\right) = -\tan \frac{\pi}{3} = -\sqrt{3}$

N: $\rightarrow F'''(x) = -\sec^2 x$

Sl: $F'''(\frac{\pi}{3}) = -4$

$\rightarrow F''''(x) = -2\sec x \cdot \tan x$

$$F''''(\frac{\pi}{3}) = -2\sec \frac{\pi}{3} \cdot \tan \frac{\pi}{3}$$

$$= -2(2)\sqrt{3}$$

$$= -4\sqrt{3}$$

$\rightarrow f(x) = \log 2 - \sqrt{3}(x - \frac{\pi}{3}) - \frac{4}{2}(x - \frac{\pi}{3})^2$
 $- \frac{4\sqrt{3}}{3}(x - \frac{\pi}{3})^3 + \dots$

2 Expand $3x^3 - 2x^2 + x - 4$ in power of $(x+2)$

By Taylor's Series,

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!} f''(a)$$

$$+ \frac{(x-a)^3}{3!} f'''(a) + \dots$$

putting $a = -2$

$$F(x+2) = F(-2) + (x+2)F'(-2) + \frac{(x+2)^2 F''(-2)}{2!} + \frac{(x+2)^3 F'''(-2)}{3!} + \dots$$

$$\begin{aligned} \rightarrow F(-2) &= 3(-2)^3 - 2(-2)^2 - 2 - 4 \\ &= -36 - 8 \\ &= -24 - 8 - 6 \\ &= -38 \end{aligned}$$

$$\begin{aligned} \rightarrow F'(x) &= 9x^2 - 4x + 1 \\ F'(-2) &= 9(-2)^2 - 4(-2) + 1 \\ &= 36 + 8 + 1 \\ &= 45 \end{aligned}$$

$$\begin{aligned} \rightarrow F''(x) &= 18x - 4 \\ F''(-2) &= 18(-2) - 4 \\ &= 32 - 40 \end{aligned}$$

$$\begin{aligned} \rightarrow F'''(x) &= 18 \\ F'''(-2) &= 18 \end{aligned}$$

$$\begin{aligned} \rightarrow F(x+2) &= -38 + 45(x+2) - \frac{40}{2}(x+2)^2 \\ &\quad + \frac{18}{6}(x+2)^3 \end{aligned}$$

$$= -38 + 45(x+2) - 20(x+2)^2 + 3(x+2)^3$$

3 Obtain Maclaurin series of $\sec x$.

By Maclaurin's Series,

$$F(x) = F(0) + x F'(0) + \frac{x^2}{2!} F''(0) + \dots$$

-> $F(x) = \sec x$

$F(0) = \sec 0 = 1$

-> $F'(x) = \sec x \cdot \tan x$

$F'(0) = \sec 0 \cdot \tan 0 = 0$

-> $F''(x) = \sec x \cdot \tan x \cdot \tan x + \sec x \cdot \sec^2 x$

$F''(0) = 1$

-> $F'''(x) = \sec x \cdot \tan x \cdot \tan^2 x + 2 \tan x \cdot \sec^2 x + 3 \sec^2 x \cdot \sec x \cdot \tan x$

$F'''(0) = 0$

-> $F^{iv}(x) = \sec x \cdot \tan^3 x + 2 \tan x \cdot \sec^2 x + 3 \sec^3 x \cdot \tan x$

-> $F^{iv}(x) = \sec x \cdot \tan x \cdot \tan^3 x + 3 \tan^2 x \cdot \sec^2 x + 2 \sec^2 x \cdot \sec x + 2 \tan x \cdot 2 \sec x \cdot \sec x \cdot \tan x + 9 \sec^2 x \cdot \tan x \cdot \sec x \cdot \tan x + 3 \sec^3 x \cdot \sec^2 x$

$F^{iv}(x) = 5$

$$\sec(x) = 1 + x(0) + \frac{x^2(1)}{2!} + \frac{x^3(0)}{3!} + \frac{5x^4}{4!} + \dots$$

$$\sec x = 1 + \frac{x^2}{2!} + \frac{5x^4}{4!} + \dots$$

$$4 \log(\sec x) = \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \dots$$

$$\text{let } \log(\sec x) = f(x)$$

By Maclurin's Series,

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2} f''(0) + \dots$$

$$\rightarrow f(0) = 0$$

$$\rightarrow f'(x) = \frac{1 \cdot \sec x \cdot \tan x}{\sec^2 x} = \tan x$$

$$f'(0) = 0$$

$$\rightarrow f''(x) = \sec^2 x = 1 + \tan^2 x$$

$$f''(0) = 1$$

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$$\rightarrow f'''(x) = 2 \sec^2 x \cdot \tan x$$

$$f'''(0) = 0$$

$$\rightarrow f^{iv}(x) = 4 \sec x \cdot \tan x \cdot \tan x + 2 \sec^2 x \cdot \sec^2 x$$

$$f^{iv}(0) = 2$$

$$\rightarrow \log(\sec x) = \frac{x^2}{2} + \frac{x^4}{12} + \dots$$

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