Unit - 2 - Discrete Random Variable

* Task : 1 : Random Variable :

1 Define Random Variable. In the experiment of tossing a fair coin three time, the sample space,

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$.

If X is the random variable giving number of heads obtained find, (a) $P(X = 2)$ (b) $P(X<2)$

=) Random Variable :

Random Variable is a variable which represt the outcome of a ~~k~~ trial an event.

=) $S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

a $P(X = 2) = \frac{3}{8}$

b $P(x < 2) = \frac{4}{8} = \frac{1}{2}$

2 Consider the experiment of throwing fair die. Let x be the random variable which assigns 1 if the number is even and 0 if the number is odd a) what is the range of x? b) $P(x = 1)$ and $P(x = 0)$

=> Sample Space

$S = \{1, 2, 3, 4, 5, 6\}$

a Here, Range $x = \{0, 1\}$ because there are only 0 and 1 possible condition. If Number is even than 1 else Number is odd than 0.

b $P(x = 1) = \frac{3}{6} = \frac{1}{2}$

$P(x = 0) = \frac{3}{6} = \frac{1}{2}$

3 A die is tossed twice, Getting 'An odd Number' is success. Find the probability of number of success and Write distribution table.

=) Let X = Random Variable which is Number of success.

A die is tossed twice so,
Range of X = {0, 1, 2}

- For X = 0, P(X=0) = Get no odd number of two die.

$P(X=0) = \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

- For P(X = 1) = P(Get odd Number only one die)

$= 2C_1 \times \frac{3}{6} \times \frac{3}{6}$

$= \frac{1}{2}$

- For P(X = 2) = P(Get odd Number on both die)

$= 2C_2 \times \frac{3}{6} \times \frac{3}{6} = \frac{1}{4}$

Distribution table:

X	0	1	2
$P(X)$	1/4	1/2	1/4

4

X	0	1	2	3	4	5	6
$P(X)$	K	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$

(a) Find $P(X<4)$, $P(X \geq 5)$, $P(3<X\leq 6)$
(b) What will be the minimum value of k so that $P(X\leq 2) > 0.3$

Here, Given that,

$$P(X \geq 2) > 0.3$$

Total Probability is 1,

$\therefore K + 3K + 5K + 7K + 9K + 11K + 13K = 1$

$\therefore \quad K = \frac{1}{49}$

(a) $P(X<4) = 1 - P(X \geq 4)$

$= 1 - P(X=5) + P(X=6) + P(X=4)$

$= 1 - (11K + 13K) + 9K$

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$P(X<4) = \cancel{1 - 24k} = 1 - \frac{33}{49}$

$\cancel{= 1 - \frac{24}{49}}$

$P(X<4) = \frac{16}{49}$

- $P(X \geq 5) = P(X=5) + P(X=6)$
$= 11k + 13k$
$= 24k$
$= \frac{24}{49}$

- $P(3 < X \leq 6) = P(X=4) + P(X=5) + P(X=6)$

$= 9k + 11k + 13k$

$= 33k = \frac{33}{49}$

(b) $P(X \leq 2) > 0.3$

$P(X=0) + P(X=1) + P(X=2) > 0.3$
$k + 5k + 3k > 0.3$
$\therefore k = \frac{1}{30}$

5 Consider the experiment of tossing coin three times. Let X be the random variable giving the number head obtained.

(a) What is Range of X ?

(b) $P(X=0)$, $P(X=1)$, $P(X=2)$, $P(X=3)$

Here, Coin is tossing three times.

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

X = Head obtained Number.

(a) Here we Head only 4 times.

So, $X = \{0, 1, 2, 3\}$

(b) $P(X=0) = P(\{TTT\})$

$= (1-P)(1-P)(1-P)$

$= (1-P)^3$

$P(X=1) = P(\{TTH, THT, HTT\})$

$= P(1-P)(1-P) + P(1-P)(1-P) + P(1-P)(1-P)$

$= 3p(1-p^2)$

- $P(X=2) = P(\{HHT, HTH, THH\})$
$= p^2(1-p) + p^2(1-p) + p^2(1-p)$
$= 3p^2(1-p)$

- $P(X=3) = P(\{HHH\})$
$= p^3$

6 Four bad apples are mixed accidently with 20 good apples. Obtain the Probability the number of bad apples in a draw of 2 apple at Random.

=> Sample Space = 24 apple.

Here, We have to draw 2 apple Random.

- Case-1 - 1 apple is bad and 1 apple is Good.

$$= \frac{4C_1 \times 20C_1}{24C_2}$$

$$= \frac{80}{276}$$

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- Case : 2 : Both Apple are bad and Zero Good apple

$$= \frac{4C_2 \times 20C_0}{24C_2}$$

$$= \frac{6}{276}$$

Total Probability $= 1 \times Case(1) + 2 \times Case(2)$

$$= \frac{80}{276} + \frac{12}{276}$$

$$= \frac{97}{276}$$

$$= \frac{1}{3}$$

* Task : 2 : Expected Value and Expectation of a Function of a Random Variable.

1 Write defination with its Formula.

(a) Mean : Mean of Random Variable is the weighted average of the possible values of Random Variable.

$$E(X) = \mu = \sum_{i=1}^{n} X_i P(X_i)$$

$$= X_1 P(x_1) + x_2 P(x_2) + \dots$$

(b) Moment : Moment of Random Variable is a expected value of a specified integer power of the deviation of the Random Variable.

(c) Variance : A Measure of spread for a distribution of a random variable that determines the degree to which the values of a random variable differ from the expected value.

$$Var(X) = E(X^2) - [E(X)]^2$$

(d) Standard Deviation: Standard Deviation is a square root of Variances.

$$\sigma_x = \sqrt{Var(X)}$$

2 Random Variable X has following Probability mass function.

$P(X = 1) = 1/2$, $P(X=2) = 1/4$,
$P(X=3) = 1/8$, $P(X = 4) = 1/8$

(a) Find and sketch the cumulative distrbuction function.

(b) $P(X \leq 1)$, $P(1 < x \leq 3)$, $P(1 \leq x \leq 3)$

(b) $P(X \leq 1) = 1 - P(X \geq 1)$
$= 1 - P(X = 2) + P(X = 3) + P(X = 1) + P(X = 4)$
$= 1 - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}\right)$

$= 1$

- $P(1 < x \leq 3) = P(X = 2) + P(X = 3)$

$= \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

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$$P(1 \leq X \leq 3) = P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$$

$$= \frac{7}{8}$$

(a)

4 A die is tossed twice. Getting a number gretor than 4 is considered as succes. Find the Mean, Variance and Standard deviation of the probability distrubuction of number of successes.

=> Here, Die is tossed twice.

- $X = 0$ = Two die does not get Greter than 4 number

$$P(X=0) = \frac{16}{36} = \frac{4}{9}$$

- $X = 1$ = Only one die get Greter than 4 Number

$$P(X=1) = \frac{16}{36} = \frac{4}{9}$$

- $X = 2$ = Only Two die get Greter than 4 Number

$$P(X=2) = \frac{4}{36} = \frac{1}{9}$$

→ Mean $= E(X) = x_0 P(x_0) + P(x_1) x_1 + x_2 P(x_2)$

$$E(x) = 0 + \frac{4}{9} + \frac{2}{9}$$

$$= \frac{2}{3}$$

→ Variance

$$E(x^2) = x_0^2 P(x_0) + x_1^2 P(x_1) + x_2^2 P(x_2)$$

$$= 0 + \frac{4}{9} + \frac{4}{9}$$

$$= \frac{8}{9}$$

$$(E(x))^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

Variance $= E(x^2) - (E(x))^2$

$$= \frac{8}{9} - \frac{4}{9}$$

$$= \frac{4}{9}$$

$\Rightarrow$ $S.D = \sqrt{Var(x)} = \sqrt{\frac{4}{9}}$

$S.D = \frac{2}{3}$

5 A random variable x is defined as the sum of the faces when a pair of dice is thrown. Find the expected value of x.

$\Rightarrow$ Let x is a sum of all the faces.

Range of $X = \{1, 2, 3, ---, 12\}$

$P(X = 2) = \{(1, 1)\} = 1/36$

$P(X = 3) = \{(1, 2), (2, 1)\} = 2/36$

$P(X = 4) = \{(2, 2), (1, 3), (3, 1)\}$
$= 3/36$

$P(X = 5) = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$
$= 4/36$

$P(X = 6) = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$
$= 5/36$

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$P(X = 7) = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$
$= 6/36$

$P(X = 8) = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$
$= 5/36$

$P(X = 9) = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$
$= 4/36$

$P(X = 10) = \{(4, 6), (6, 4), (5, 5)\}$
$= 3/36$

$P(X = 11) = \{(6, 5), (5, 6)\} = 2/36$

$P(X = 12) = \{(6, 6)\} = 1/36$

$\rightarrow E(X) = \sum_{i=2}^{12} P(X_i) \cdot x_i$

$$= \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$

$$= \frac{252}{36}$$

$= 7$

6 Let X denote a random variable that takes on any of the Values -1, 0, 1 with respective probabilities $P(X=-1)=0.2$, $P(X=0)=0.5$, $P(X=1)=0.3$ Compute $E(X^2)$.

=> Here Given, $P(X=-1)=0.2$,
$P(X=0)=0.5$,
$P(X=1)=0.3$

$$E(X^2) = x_0^2 \cdot P(x_0) + x_1^2 \cdot P(x_1) + x_{(-1)}^2 \cdot P(x_{(-1)})$$

$$= (-1)(0.2) + 0(0.5) + 1(0.3)$$

$$= 0.5$$

7 A School class of 120 Students is driven in 3 buses to a symphonic performance. There are 36 student in one of the buses, 40 in another and 44 in the third bus. When the buses arrive, one of the 120 Students is randomly chosen. Let X denotes the number of Students on the bus of

that randomly chosen student and find $E[x]$.

=) Here Given that,

$$P(X=36) = \frac{36}{120}$$

$$P(X=40) = \frac{40}{120}$$

$$P(X=44) = \frac{44}{120}$$

$$E(X) = 36 \times \frac{36}{120} + 40 \times \frac{40}{120} + 44 \times \frac{44}{120}$$

$$= \frac{4832}{120}$$

$$= 40.2667$$

8 A die is tossed thrice. Getting 1 or 6 on a toss is a success. Find the mean or expectation and Variance.

=) Total Number of Trial = 3

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Total ~~fervous~~ faliure = 2

Probability of success $= \frac{2}{6} = \frac{1}{3}$

Mean $= n \cdot p = 3 \times \frac{1}{3} = 1$

$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Variance = Mean $\cdot$ q

$= 1 \cdot \frac{2}{3}$

$= \frac{2}{3}$

* Task : 3 : Binomial Distribution.

1 A die is tossed 3 times. What is the Probability of
(a) No Five turnning up?
(b) 1 five?
(c) 3 five?

=> No. of Trial = 3
r = No. of Five turnning.

$p = \frac{1}{6}$, $q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

(a) $r = 0$

$$P(r = 0) = nC_r \cdot p^r \cdot q^{n-r}$$

$$= 3C_0 \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{5}{6}\right)^3$$

$$= 0.5787$$

(b) $r = 1$

$$P(r = 1) = nC_r \cdot p^r \cdot q^{n-r}$$

$$= 3C_1 \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^2$$

$$= 0.3472$$

(c) $r = 3$

$$P(r=3) = 3C_3 \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^0$$

$$= 4.62 \times 10^{-3}$$

2 Hospital records show that of patients suffering from a certain disease 75 % die of it. What is the Probability that of 6 randomanly selected Patients, 4 will recover?

=) No. of Trial = 6
No. of Patients will recover = r
So $r = 4$

Given $q = 0.75$
$p = 1 - q = 0.25$

$$P(X=4) = nC_r \cdot p^r \cdot q^{n-r}$$

$$= 6C_4 \cdot (0.25)^4 \cdot (0.75)^2$$

$$= 0.0329$$

3 In 256 sets of 12 tosses of a coin, in how many cases one can expect 8 heads and 4 tails.

=> Number of Trials = 12

$P(H) = 0.5$, $P(T) = 0.5$

$P(X=8) = 12C_8 (0.5)^8 \cdot (0.5)^4$

$= 0.12084$

Mean = $256 \times P(X=8)$

$= 256 \times 0.12084$

$= 30.9$

4 A marksman finds that on the average he hits the target 4 times out of 5. If he shots, what is the probability of
(a) More than 2 hits?
(b) at least 3 Misses?
(c) 3 Fives?

=> No. of Trial = 4

$P = \frac{4}{5}$, $q = 1 - P = 1 - \frac{4}{5} = \frac{1}{5}$

(a) $P(x) = P(x_3) + P(x_4)$

$= 4C_3 \cdot (0.8)^3 \cdot (0.2) +$

$4C_4 \cdot (0.8)^4 \cdot (0.2)^0$

$= 0.8192$

(b) Atleast 3 Means 3 misses and 1 hits.

$P(x) = 4C_1 \cdot (0.8)^1 \cdot (0.2)^3 +$

$4C_0 \cdot (0.8)^0 \cdot (0.2)^4$

$= 0.0272$

5 Find the number of a coin that are needed so that the probability of getting atleast one head is 0.875

=> $P = 0.875 =$ Atleast One head Probability

Total Probability,

$P + q = 1$

After tossing n times coin
Probability of $q = 0.5^n$

$\therefore 0.875 + 0.5^n = 1$

$\therefore n = \log_{0.5}(1 - 0.875)$

$\therefore n = 3$

6 A manufacturer of metal pistons finds that on the average 12% of hits pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain
(a) No more than 2 rejects?
(b) At least 2 rejects?

=> $n = 10$ Pistons
X = No. of Reject Pistons
$p = 0.12$
$q = 1 - p = 1 - 0.12 = 0.88$

(a) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 10C_0 \cdot (0.12)^0 (0.88)^{10} +$

$10C_1 \cdot (0.12)^1 (0.88)^9 + 10C_2 \cdot (0.12)^2 (0.88)^8$

$P(X \leq 2) = 0.891$

(b) $P(X \geq 2) = 1 - P(x < 2)$

$= 1 - P(X=0) + P(X=1)$

$= 1 - 0.2785 + 0.3793$

$= 0.3417$

7 In sampling of a large number of parts manufactured by a machine, the mean number of defective in a sample of 20 is 2. Out of 100 such sample. Find the Probability of atleast three defective in a sample of 20 and then how many would be expected to contain atleast 3 defective parts.

$\Rightarrow$ $n = 20$, Mean $= 2$

Mean $= n \times P$

$\therefore P = \frac{2}{20} = 0.1$

$q = 1 - P = 1 - 0.1 = 0.9$

$P(x \geq 3) = 1 - P(x < 3)$

$= 1 - P(0) + P(1) + P(2)$

$= 1 - 20C_0 (0.1)^0 (0.9)^{20} +$

$20C_1 (0.1)^1 (0.9)^{19} + 20C_2 (0.2)^2 (0.9)^{18}$

$P(X \geq 3) = 0.323$

For 1000 Sample $= 1000 \times 0.323$
$= 323$

8 With the usual notations. Find p for a binomial random variable X if n=6 and if $9P(X=4) = P(X=2)$

$\Rightarrow$ $n = 6$

$\therefore 9P(X=4) = P(X=2)$

$\therefore 9 \cdot 6C_4 \cdot p^4 (1-p)^2 = 6C_2 \cdot p^2 (1-p)^4$

$\therefore 9p^2 = (1-p)^2$

$\therefore 3p = (1-p)$

$\therefore p = \frac{1}{4}$

* Task : 4 : Poison Distribution

1 A life insurance salesman sells on the average 3 life insurance policies per week. Use Poison's low to calculate the probability that in a given week he will sell

(a) Some Policies

(b) 2 or more policies but less than 5 policices.

(c) 3 Assumming that three are 5 working days per week. What is the Probability that in a given day he will sell one policy ?

=> Given $\lambda = 3$

(a) $P(x > 0) = 1 - P(X = 0)$

$$= 1 - \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= 1 - \frac{e^{-3} \cdot 3^{0}}{0!}$$

$$= 1 - 0.049$$

$$= 0.950$$

(b) $P(2 \le x \le 5) = P(2) + P(3) + P(4)$

$$= \frac{e^{-3} \cdot \lambda^2}{2!} + \frac{e^{-3} \cdot \lambda^3}{3!} + \frac{e^{-3} \cdot \lambda^4}{4!}$$

$$= 0.616$$

(6) Avg. Policy Per day $= \frac{3}{5} = 0.6$

$$P(x) = \frac{e^{-0.6} \cdot (0.6)^1}{1!}$$

$$= 0.329$$

2 It is known from the past experiance that in a certain plant there are on the average 4 industrial accidents per month. Find the probability that in a given year there will be less than 4 accidents. $(e^{-4} = 0.0183)$.

=> $m = 4$

$P(x < 4) = P(0) + P(1) + P(2) + P(3)$

$$= \sum_{x=0}^{3} \frac{e^{-4} \cdot 4^x}{x!}$$

$$= e^{-4}\left[\frac{4^0}{0!} + \frac{4}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!}\right]$$

$$= 0.0183\,(71/3)$$

$$= 0.4331$$

3 Between the hours 2 pm and 4 pm, the average number of phone calls per minute coming into the switch boards of a company is 2.35. Find the Probability that during one particular minute, there will be at most 2 phone calls $(e^{-2.35} = 0.0953)$

$\Rightarrow$ Mean = m = 2.35

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= e^{-2.35}\left(\frac{2.35^0}{0!} + \frac{2.35^1}{1!} + \frac{2.35^2}{2!}\right)$$

$$= 0.0953\,(6.11)$$

$$= 0.5828$$

4 In a certain factory turning out razor blades, there is a small chance of 0.002 of any blades to be defective. The blades are supplied in packets of 10. Use possion distribuction to calculate the approximate number of packets containg no defective, one defective and two defective blades of 10000 packets.

=> Number of Packets $n = 10$

Mean $= \lambda = 0.02$

- $P(X=0) = \dfrac{e^{-0.02} \times (0.02)^0}{0!}$

$= 0.9802$

For 10000 Packets $= 1000 \times 0.9802$

$= 9802$

- $P(X=1) = \dfrac{e^{-0.02} \times (0.02)^1}{1!}$

$= 0.01960$

For 10000 Packets $= 10000 \times 0.01960$

$= 196$

- $P(x=2) = \dfrac{e^{-0.02} \times (0.02)^2}{2!}$

$= 0.000196$

For 10000 Packet $= 10000 \times 0.000196$

$= 2$

5 There are five students in a class and the number of students who will participate in annual day every year is a mean 3. What will the Probability of more than 3 students ?

=> Mean $= \lambda = 3$

$P(x > 3) = P(4) + P(5)$

$= \dfrac{e^{-3} \cdot 3^4}{4!} + \dfrac{e^{-5} \cdot 3^5}{5!}$

$= e^{-3} [3.375 + 2.025]$

$= 0.04978 [5.4]$

$= 0.2688$

6 The deals cracked by an agent per day is a random variable with mean 2. Given that each day is independent of other day. Find the probability getting 2 deals cracked on First day and 1 deals cracked on next day.

=> Mean = $\lambda = 2$

Probability = $P(2,2) + P(1,2)$

$$= \frac{e^{-2} \cdot 2^2}{2!} + \frac{e^{-2} \cdot 2^1}{1!}$$

$$= e^{-2}\left[4/2 + 2\right]$$

$$= 0.135 \times 4$$

$$= 0.54134$$

7 Suppose a Fast Food restaurant can expect two customers every 3 minutes on average. What is the Probability that four or fewer patrons will enter the restaurant in a 9 minutes period?

=> Food Restaurant, two customer visit every 3 min. means 6 Customer visit every 9 min

$$P(X \le 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \frac{e^{-6} \cdot 6^0}{0!} + \frac{e^{-6} \cdot 6^1}{1!} + \frac{e^{-6} \cdot 6^2}{2!} +$$

$$\frac{e^{-6} \cdot 6^3}{3!} + \frac{e^{-6} \cdot 6^4}{4!} \cancel{+ 6}$$

$$= e^{-6}\left[1 + \frac{6}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!} + \frac{6^4}{4!}\right]$$

$$= e^{-6}[115]$$

$$= 0.00247 \times 115$$

$$= 0.285$$