

Unit : 5 : Interpolation

* Task : 1 : Relation Between Operators.

1 Prove that $(1 + \Delta)(1 - \nabla) = 1$

$$\Rightarrow \text{L.H.S.} = (1 + \Delta)(1 - \nabla)$$

We know $1 + \Delta = E$ and
 $1 - \nabla = E^{-1}$

$$= E \cdot E^{-1}$$

$$= 1$$

$$\text{L.H.S.} = \text{R.H.S.}$$

2 Prove that $\delta = 2 \sinh\left(\frac{hD}{2}\right)$

We know that, $E' = e^{hD}$

$$\delta f(x) = f(x + h/2) - f(x - h/2)$$

$$= E^{1/2} f(x) - E^{-1/2} f(x)$$

$$= f(x) \left[E^{1/2} - E^{-1/2} \right]$$

$$\delta F(x) = F(x) \left(e^{hD/2} - e^{-hD/2} \right)$$

$$\delta = 2 \sinh \left(\frac{hD}{2} \right) = 2 \left(\frac{e^{hD/2} - e^{-hD/2}}{2} \right)$$

$$\left(\because \sinh x = \frac{e^x - e^{-x}}{2} \right)$$

Hence, proved $\delta = 2 \sinh \left(\frac{hD}{2} \right)$

3 Prove that $\Delta \log F(x) = \log \left(1 + \frac{\Delta F(x)}{F(x)} \right)$

$$\Rightarrow \text{L.H.S.} = \Delta \log F(x)$$

$$= \log F(x+h) - \log F(x)$$

$$= \log \left(\frac{F(x+h)}{F(x)} \right)$$

$$= \log \left((1 + \Delta) \frac{F(x)}{F(x)} \right)$$

$$= \log \left(1 + \frac{\Delta F(x)}{F(x)} \right)$$

$$\text{L.H.S.} = \text{R.H.S.}$$

4 Prove that $\Delta \nabla = (\Delta - \nabla)$

\Rightarrow L.H.S. = $\Delta \nabla$

$$= (E - 1)(1 - E^{-1})$$

$$= E - EE^{-1} - 1 + E^{-1}$$

$$= E - 1 - 1 + E^{-1}$$

$$= \Delta - \nabla$$

L.H.S. = R.H.S.

5 Prove that $\Delta = E \nabla = \Delta E$

\Rightarrow We know that,

$$\Delta = 1 - E^{-1}$$

$$\Delta = 1 - \frac{1}{E}$$

~~$$\therefore \Delta E = E \times 1 - 1$$~~

$$\therefore \Delta = \frac{E - 1}{E}$$

$$\therefore \Delta E = E - 1 \quad \text{--- (1)}$$

$$\therefore \nabla E = (1 - E^{-1})E = E - 1 \quad \text{--- (2)}$$

$$\therefore \Delta = E - 1 \quad \text{--- (3)}$$

$$e_0^n \quad 1 = 2 = 3$$

$$\Delta = E\Delta = E\nabla$$

6 Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$

$$\Rightarrow \text{R.H.S.} = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$$

$$= \frac{\Delta^2 - \nabla^2}{\nabla\Delta}$$

$$= \frac{(\Delta - \nabla)(\Delta + \nabla)}{(\Delta - \nabla)}$$

$$= (\Delta + \nabla)$$

$$\text{R.H.S.} = \text{L.H.S.}$$

7 Find the missing terms in the table.

(a)

X	0	1	2	3	4
Y	1	3	9	?	81

X	Y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	1				
		2			
1	3		4		
		6		$a-19$	
2	9		$a-15$		$124-4a$
		$a-9$		$105-3a$	
3	a		$90-2a$		
		$81-a$			
4	81				

$\therefore \Delta^4 y = 0$

$\therefore 124 - 4a = 0$

$\therefore a = \frac{124}{4} = 31$

(6)

X	2	2.1	2.2	2.3	2.4	2.5	2.6
Y	0.135	Y_2	0.111	0.1	Y_5	0.08	0.074

Missing Term $Y_2 = a$

$Y_5 = b$

X	Y	ΔY	ΔY^2	ΔY^3	ΔY^4	ΔY^5
2	0.135					
		$a - 0.135$				
2.1	a		$0.246 - 2a$			
		$0.111 - a$		$-0.368 + a$		
2.2	0.111		$-0.122 - a$		0.401	
		-0.011		$0.033 + b - a$	$-2a$	
2.3	0.1		$b - 0.089$		$0.234 - 4b$	
		$b - 0.1$		$0.269 - 3b$	$+a$	
2.4	b		$0.18 - 2b$		-0.535	
		$0.08 - b$		$-0.266 - 3b$	$+6b$	
2.5	0.08		$-0.086 - b$			
		-0.006				
2.6	0.074					

* Task : 2 : M Newton Gregory Forward and Backward Interpolation.

1 Find Value of $\sin(52^\circ)$ from table.

θ°	45°	50°	55°	60°
$\sin(\theta^\circ)$	0.7071	0.7660	0.8192	0.8660

=> For Finding the value of $\sin(52^\circ)$ we have to use Newton Forward Interpolation,
So, $x = 52^\circ$

$$\therefore p = \frac{x - x_0}{h} = \frac{52 - 45}{5} = 1.4$$

θ°	$\sin \theta^\circ$	Δy	$\Delta^2 y$	$\Delta^3 y$
45	0.7071			
		0.0589		
50	0.7660		-0.0057	
		0.0532		-0.0007
55	0.8192		-0.0064	
		0.0468		
60	0.8660			

$$y = y_0 + p \Delta y + \frac{p(p-1)}{2!} \Delta^2 y + \frac{p(p-1)(p-2)}{3!} \Delta^3 y$$

$$y = 0.7071 + 1.4(0.0584) + \frac{1.4(1.4-1)}{2!} (-0.0057) + \frac{1.4(1.4-1)(1.4-2)}{3!} (-0.0007)$$

$$y = 0.7071 + 0.08246 + 0.001546 + 0.0000392$$

$$y = 0.7880$$

Value of $\sin(52^\circ) = 0.7880$

2. Compute $\cosh(0.56)$ Using Newton's forward difference

x	0.5	0.6	0.7	0.8
$\cosh(x)$	1.12762	1.18546	1.25516	1.33743

\Rightarrow For Finding the value of $\cosh(0.56)$ we have to use Newton's Forward Interpolation,

$$\text{So, } x = 0.56 \text{ and } x_0 = 0.5$$

$$\therefore p = \frac{x - x_0}{h} = \frac{0.56 - 0.5}{0.1} = 0.6$$

x	$\cosh(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
0.5	1.1276			
		0.057839		
0.6	1.1854		0.011865	
		0.069704		-0.000667
0.7	1.2551		0.012562	
		0.082266		
0.8	1.3374			

$$y = Y_0 + P\Delta y + \frac{P(P-1)}{2!} \Delta^2 y + \frac{P(P-1)(P-2)}{3!} \Delta^3 y$$

$$y = 1.127626 + 0.0347034 + (-0.0014238) + (-0.0000390)$$

$$y = 1.1608665$$

4 Find value of y for $x = 21$ and $x = 28$ from the following.

x	20	23	26	29
y	0.3420	0.3907	0.4384	0.4848

X	Y			
20	0.3420			
		0.0487		
23	0.3907		-0.0010	
		0.0477		-0.00030
26	0.4384		-0.0013	
		0.0464		
29	0.4848			

\Rightarrow For Finding the value of $x = 21$
we have to use Newton's
Forward Interpolation.

$$\text{So, } x = 21, \quad x_0 = 20$$

$$p = \frac{x_0 - x_0}{h} = \frac{21 - 20}{3} = 0.3333$$

$$Y = Y_0 + p \Delta Y + \frac{p(p-1)}{2!} \Delta^2 Y +$$

$$\frac{p(p-1)(p-2)}{3!} \Delta^3 Y$$

$$Y = 0.3420 + 0.01623 + 0.000111 - 0.0000185$$

$$Y = 0.3583$$

⇒ For Finding the value of $x = 28$, we have to use Newton's Backward Interpolation.

So, $x = 28$, $x_3 = 29$

$$p = \frac{x - x_3}{h} = \frac{28 - 29}{3} = (-0.3333)$$

$$y = y_3 + p \nabla y + \frac{p(p+1)}{2!} \nabla^2 y +$$

$$\frac{p(p+1)(p+2)}{3!} \nabla^3 y$$

$$y = 0.4848 + (-0.01546) + 0.000144$$

$$y = 0.4695$$

5 The area A of a circle of diameter d is given for the following values and calculate the area of a circle of a diameter of 105 unit using Newton's backward formula.

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

=> For Finding the value of 105,
We have to use Newton's
Backward Interpolation.

$$\text{So, } x = 105, \quad x_n = 100$$

$$p = \frac{x_m - x_n}{h} = \frac{105 - 100}{5} = 1$$

d	A	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
80	5026				
		648			
85	5674		40		
		688		-2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
100	7854				

$$y = y_n + p \nabla y + \frac{p(p+1)}{2!} \nabla^2 y +$$

$$\frac{p(p+1)(p+2)}{3!} \nabla^3 y + \frac{p(p+1)(p+2)(p+3)}{4!} \nabla^4 y$$

$$y = 7854 + 766 + 40 + 2 + 4$$

$$= 8660$$

3 From the following data, Find the number of person earning weekly wages between 100 and 110 rupess.

Wages	Below-80	80-100	100-120	120-140	140-160
No. of Person	250	120	100	70	50

⇒ For Finding the value of wages between 100 and 110, we have to use Newton's Forword Interpaletion.

So, $x = 110$, $x_0 = 80$

∴ $P = \frac{110 - 80}{20} = 1.5$

X	Y	ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$
80	250				
		120			
100	370		-20		
		100		-10	
120	470		-30		20
	540	70		10	
140	70		-20		
	590	50			
160	50				

$$y = y_0 + P \Delta y_0 + \frac{P(P-1)}{2!} \Delta^2 y_0 +$$

$$\frac{P(P-1)(P-2)}{3!} \Delta^3 y_0 +$$

$$\frac{P(P-1)(P-2)(P-3)}{4!} \Delta^4 y_0$$

$$y = 250 + (1.5)(120) + 1.5(1.5-1)$$

$$\frac{(1.5-2)(-20)}{2!} + 1.5(1.5-1)$$

$$\frac{(1.5-2)(\cancel{1.5-2})(\cancel{-20})(-10)}{3!} +$$

$$\frac{(1.5)(1.5-1)(1.5-2)(1.5-3)(20)}{4!}$$

$$y = 422.65625$$

Here, 422.65625 weekly wages 110 Rs. and

370 weekly wages below 100 Rs.

So, Between 100 and 170
rupess

Weekly = 422.65625 - 370
wages =

= 52.594

* Task : 3 : Gauss Forward and Backward Interpolation.

1 Using the Gauss's Forward to get $f(6.5)$ for following data.

x	4	5	6	7	8	9
$f(x)$	14.141	12.043	10.223	8.647	7.260	6.045

\Rightarrow For Finding the value of $x = 6.5$ we have to use Gauss's Forward Method,

So, $x = 6.5$ and $x_0 = 6$

$$P = \frac{x - x_0}{h} = \frac{6.5 - 6}{1} = 0.5$$

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
4	14.141					
		-2.098				
5	12.043		0.278			
		-1.82		-0.034		
6	10.223		0.244		0.089	
6.5		-1.576		-0.055		-0.012
7	8.647		0.189		0.077	
		-1.387		-0.017		
8	7.260		0.172			
		-1.215				
9	6.045					

$$\begin{aligned}
 Y = Y_0 + P(-1.576) + \frac{P(P-1)(0.244)}{2!} \\
 + \frac{P(P-1)(P+1)(-0.055)}{3!} + \\
 \frac{P(P-1)(P-2)(P+1)(0.089)}{4!} + \\
 \frac{P(P-1)(P-2)(P+1)(P+2)(-0.012)}{5!}
 \end{aligned}$$

$$Y = 9.398$$

2 Using Gauss's forward interpolation formula, find the population of the year 1986 for data.

Year	1961	1971	1981	1991	2001	2011
Population	14	17	32	43	60	95

=> For finding the value of 1986, we have to use Gauss's forward interpolation.

$$\text{So, } x = 1986, x_0 = 1981$$

$$\therefore p = \frac{x - x_0}{h} = \frac{1986 - 1981}{10} = 0.5$$

Year	Population	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1961	14				
		3			
1971	17		12		
		15		-8	
1981	32		4		
1986		11		2	
					10
1991	43		6		10
		17		12	
2001	60		18		
		35			
2011	95				

$$Y = 32 + 0.5(11) + \frac{(0.5)(0.5-1)}{2!} 4$$

$$+ \frac{0.5(0.5-1)(0.5+1)}{3!} 2 +$$

$$\frac{0.5(0.5-1)(0.5+1)(0.5-2)}{4!} 10$$

$$Y = 37.0683$$

3 Use Gauss's Forward Interpolation Formula to find 3.3 from the following data.

x	1	2	3	4	5
y	15.3	15.1	15	14.5	14

=> For finding the value of 3.3, we have to use Gauss's Forward Interpolation.

So, $x = 3.3$ and $x_0 = 3$

$$p = \frac{x - x_0}{h} = \frac{3.3 - 3}{1} = 0.3$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	15.3				
		-0.2			
2	15.1		0.1		
		-0.1		-0.5	
3	15		-0.4		0.9
3.3		-0.5		0.4	
4	14.5		0		
		-0.5			
5	14				

$$Y = 15 + 0.3(-0.5) + \frac{(-0.5)(-0.5-1)(-0.4)}{2!}$$

$$+ \frac{(-0.5)(-0.5-1)(-0.5+1)(0.4)}{3!} +$$

$$\frac{(-0.5)(-0.5-1)(-0.5-2)(-0.5+1)(0.9)}{4!}$$

$$Y = 14.89$$

4 Find $Y(2.36)$ from the following table by using Gauss's Backward Formula.

x	1.6	1.8	2	2.2	2.4	2.6
y	4.95	6.05	7.89	9.03	11.02	13.46

\Rightarrow For finding the value of 2.36, we have to use Gauss's Backward Formula.

$$\text{So, } x = 2.36 \text{ and } x_0 = 2.2$$

$$h = \frac{x - x_0}{h} = \frac{2.36 - 2.2}{0.2} = 0.8$$

X	Y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1.6	4.95					
		1.1				
1.8	6.05		0.24			
		1.34		0.06		
2	7.39		0.3		-0.02	
		1.64		0.04		0.09
2.2	9.03		0.34		0.07	
2.36		1.99		0.11		
2.4	11.02		0.45			
		2.44				
2.6	13.46					

$$Y = 9.03 + 0.8(1.64) + \frac{0.8(0.8+1)}{2!}$$

$$+ \frac{0.34 + 0.8(0.8+1)(0.8-1)0.04}{3!}$$

$$+ \frac{0.8(0.8+1)(0.8-1)(0.8+2)0.07}{4!}$$

$$+ \frac{0.8(0.8+1)(0.8-1)(0.8+2)(0.8-2)0.09}{5!}$$

$$Y = 10.62$$

5 From the following table, Find y when $x=38$.

x	30	35	40	45	50
y	15.9	14.9	14.1	13.3	12.6

\Rightarrow For Finding the value of 38 we have to use Gauss's Forward Interpolation,

So, $x = 38$ and $x_0 = 35$

$$p = \frac{x - x_0}{h} = \frac{38 - 35}{5} = 0.6$$

X	Y	ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$
30	15.9	-1			
35	14.9	-0.8	0.2		
38				-0.2	
40	14.1	-0.8	0		-0.2
45	13.3	-0.8	0	0	
50	12.5				

$$Y = 4.9 + P(-0.8) + \frac{P(P-1)}{2!} 0.2$$

$$+ \frac{P(P-1)(P+1)}{3!} (-0.2)$$

~~$$Y = 14.9 - 0.48 - 0.024 - 0.01$$~~

$$Y = 14.413$$

* Task: 4: Stirling's and Bessel's Interpolation.

2 Using Stirling's interpolation Formula, to compute $Y(35)$ from the following data.

x	20	30	40	50
Y	512	439	346	243

\Rightarrow For Finding the value of $Y(35)$ we have to use Stirling's interpolation.

So, $x_0 = 35$ and $x_1 = 30$

So,

$$p = \frac{x - x_0}{h} = \frac{35 - 30}{10} = 0.5$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	512			
		-73		
30	439	-73	-20	
35		-93		10
40	346		-10	
		-103		
50	243			

$$Y = Y_0 + P \frac{(-73 - 93)}{2} + \frac{P^2}{2!} (-20)$$

$$= 439 + 0.5 \frac{(-73 - 93)}{2} + \frac{(0.5)^2}{2!} (-20)$$

$$= 439 - 41.5 - 2.5$$

$$= 395$$

3 Use Bessel's interpolation formula to find $Y(25)$ from the following data.

x	20	24	28	32
y	2854	3162	3544	3992

For finding the value of $Y(25)$ we have to use Bessel's Interpolation.

$$\text{So, } x = 25, \quad x_0 = 24$$

$$\therefore P = \frac{x - x_0}{h} = \frac{25 - 24}{4}$$

$$P = 0.25$$

X	Y	ΔY	$\Delta^2 Y$	$\Delta^3 Y$
20	2854			
		308		
24	3162		74	
25	3382			-8
28	3544		66	
		448		
32	3992			

$$y = \frac{1}{2} (Y_0 + Y_1) + \frac{1}{2} (2p-1) \Delta Y_0 +$$

$$\frac{(2p-1)(2p-3)}{2!} \left(\frac{\Delta^2 Y_{-1} + \Delta^2 Y_0}{2} \right) +$$

$$\frac{(2p-1)(2p-3)(2p-5)}{3!} \Delta^3 Y_{-1}$$

$$y = 3353 - 95.5 - 6.5625 - 0.0625$$

$$= 3250.875$$

4 Find $Y(3.75)$ from the following table by using Bessel's interpolation formula.

X	2.5	3	3.5	4	4.5
Y	24.145	22.043	20.225	18.644	17.262
	5				
	16.047				

⇒ For Finding the value of $Y(3.75)$ we have to use Bessel's interpolation method.

So, $x = 3.75$, $x_0 = 3.5$

$$P = \frac{x - x_0}{h} = \frac{3.75 - 3.5}{0.5} = 0.5$$

X	Y	ΔY	$\Delta^2 Y$	$\Delta^3 Y$	$\Delta^4 Y$	$\Delta^5 Y$
2.5	24.145					
		-2.102				
3	22.043		0.284			
		-1.818		-0.047		
3.5	20.225		0.237		0.009	
3.75	↑	-1.581	↑	-0.038	↑	-0.002
4	18.644		0.199		0.006	
		-1.382		-0.032		
4.5	17.262		0.167			
		-1.215				
5	16.047					

$$\begin{aligned}
 y &= \frac{(19.644 + 20.225)}{2} + \frac{1}{2}(2P-1) \cdot \\
 &\quad (-1.582) + \frac{P(P-1)}{2!}(0.237 + 0.199) \\
 &\quad + \frac{(2P-1)(P-1)P}{3!}(-0.038) + \\
 &\quad \frac{(P+1)P(P-1)(P-2)}{4!}(0.009 + 0.005) \\
 &\quad + \frac{(2P-1)(P-1)(P-2)(P+1)P}{5!}(-0.0003)
 \end{aligned}$$

$$\begin{aligned}
 y &= 19.4345 + 0 + (-0.0545) + 0 \\
 &\quad + 0.0000585
 \end{aligned}$$

$$y = 19.38$$

1 Using the Stirling's interpolation formula to get $\tan(16^\circ)$ for following data.

x	0°	5°	10°	15°	20°	25°	30°
$\tan x$	0	0.0875	0.1763	0.2679	0.3640	0.4663	0.5774

=> For Finding the value of function we have to use, Stirling interpolation formula.

So, $x = 16$, $x_0 = 15$

$$h = \frac{x - x_0}{5} = \frac{16 - 15}{5} = 0.2$$

X	tan x	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
0	0.0875						
		0.0875					
5	0.0875		-0.0041				
		0.0916		0.0069			
10	0.1763		-0.0028		-0.0032		
		0.0888		0.0101		-0.0008	
15	0.2679		0.0073		-0.0112		0.0229
x_0		0.0961		-0.0011		0.0149	
20	0.3640		0.0062		0.0037		
		0.1023		0.0026			
2.5	0.4663		0.0088				
		0.1111					
30	0.5774						

$$y = 0.2679 + \frac{0.2(0.0888 + 0.0961)}{2}$$

$$+ \frac{(0.2)^2(0.0073)}{2!} + \frac{0.2(0.2^2 - 1)}{3!}$$

$$\frac{(0.0101 + 0.0011)}{2} + \frac{0.2^2(0.2^2 - 1)(0.0112)}{4!}$$

$$+ \frac{(0.2)(0.2^2 - 1)(0.2^2 - 4)(-0.008 - 0.0149)}{5!}$$

$$+ \frac{(0.2)^2(0.2^2 - 1)(0.2^2 - 2^2)(0.0229)}{6!}$$

$$y = 0.2679 + 0.01849 + 0.00014 -$$

$$0.000144 + 0.00001792 +$$

$$0.00002185 + 0.000004836$$

$$y = 0.28643$$

* Task : 5 : Langrange's and Divided Difference Interpolation.

1 Determine the interpolating polynomial of degree three using Langrange's interpolation For the following Table.

x	-1	0	1	3
y	2	1	0	-1

=> Here, we want to evaluate interpolation using Langrange's interpolation,

$$\text{So, } x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 3$$

$$y_0 = 2, y_1 = 1, y_2 = 0, y_3 = -1$$

By Langrange's Interpolation,

$$y = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \cdot y_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \cdot y_2 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \cdot y_1 +$$

$$\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0$$

$$y = \frac{(x+1)(x-0)(x-1)(-1)}{(3+1)(3-0)(3-1)} +$$

$$\frac{(x+1)(x-0)(x-3)(0)}{(1+1)(1-0)(1-3)} +$$

$$\frac{(x+1)(x-1)(x-3)(1)}{(0+1)(0-1)(0-3)} +$$

$$\frac{(x-0)(x-1)(x-3)(2)}{(-1-0)(-1-1)(-1-3)}$$

$$y = \frac{x(x^2-4x+3)}{-4} + \frac{(x^2-1)(x-3)}{3}$$

$$+ \frac{x(x^2-1)}{24}$$

$$y = \frac{1}{24} [-6(x^3-4x^2+3x) + x^3 - x + 8(x^3-3x^2-x+3)]$$

$$y = \frac{1}{24} [x^3 - 25x + 24]$$

2 Use Lagrange's Formula to fit a polynomial to the data given below.

X	-1	0	2	3
Y	8	3	1	12

⇒ Here, we want to evaluate interpolation using Lagrange's interpolation,

$$\text{So, } x_0 = -1, x_1 = 0, x_2 = 2, x_3 = 3$$

$$y_0 = 8, y_1 = 3, y_2 = 1, y_3 = 12$$

By Lagrange's interpolation,

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2$$

$$y = \frac{(x-0)(x-2)(x-3)(8)}{(8-0)(8-2)(8-3)} +$$

$$\frac{(x+1)(x-0)(x-3)1}{(1+1)(1-0)(1-3)} +$$

$$\frac{(x+1)(x-2)(x-3)3}{(3+1)(3-2)(3-3)} +$$

$$\frac{(x+1)(x-0)(x-2)12}{(12+1)(12-0)(12-2)}$$

$$y = \frac{x(x-2)(x-3)}{80} + \frac{x(x+1)(x-3)}{(-4)}$$

$$+ \frac{3(x+1)(x-2)(x-3)}{0} +$$

$$\frac{x(x+1)(x-2)}{130}$$

$$y = \frac{1}{6} (4x^3 + 4x^2 - 30x + 18)$$

4 Compute $f(8)$ from the following table by using Newton's Divided difference formula.

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

\Rightarrow Here, we want to find value of $f(8)$, for that we have to use Newton's Divided difference Method,

x	y	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
4	48					
		52				
5	100		15			
		97		1		
7	294		21		0	
		202		1		0
10	900		27		0	
		310		1		
11	1210		33			
		489				
13	2028					

$$\begin{aligned}
 y = & f(x_0) + (x-x_0)f(x_0, x_1) + \\
 & (x-x_0)(x-x_1)f(x_0, x_1, x_2) + \\
 & (x-x_0)(x-x_1)(x-x_2)f(x_0, x_1, x_2, x_3) \\
 & + (x-x_0)(x-x_1)(x-x_2)(x-x_3) \\
 & f(x_0, x_1, x_2, x_3, x_4)
 \end{aligned}$$

$$\begin{aligned}
 y = & 48 + (8-4)(52) + (8-4)(8-5)15 \\
 & + (8-4)(8-5)(8-7)1 + 0 + 0
 \end{aligned}$$

$$y = 448$$

5 Using Newton's divided difference interpolation formula compute $f(9.2)$ from the following data.

x	8	9	9.5	11
$f(x)$	2.079	2.197	2.251	2.397

\Rightarrow Here, we want to find value of $f(9.2)$ for that we have to use Newton's Divided Difference Method,

X	F(x)	$\Delta F(x)$	$\Delta^2 F(x)$	$\Delta^3 F(x)$
8	2.0794			
		0.1178		
9	2.1972		-0.00653	
		0.108		0.00046
9.5	2.2512		-0.00515	
		0.0977		
11	2.3978			

$$Y = F(x_0) + (x - x_0) F(x_0, x_1) + \\ (x - x_0)(x - x_1) F(x_0, x_1, x_2) + \\ (x - x_0)(x - x_1)(x - x_2) F(x_0, x_1, x_2, \\ x_3)$$

$$Y = 2.0794 + (9.2 - 8) 0.1178 + \\ (9.2 - 8)(9.2 - 9) (-0.00653) + \\ (9.2 - 8)(9.2 - 9)(9.2 - 9.5) (0.00046)$$

$$Y = 2.0794 + 0.14136 - 0.001567 \\ - 0.00003312$$

$$Y = 2.2192$$

3 Express the function $\frac{3x^2 - 12x + 12}{(x-1)(x-2)(x-3)}$

as a sum of partial function using Lagrange's formula.

=> Here, Given that

$$Y = \frac{3x^2 - 12x + 12}{(x-1)(x-2)(x-3)}$$

$$\text{So, } x_0 = 1, x_1 = 2, x_2 = 3$$

By Lagrange's formula,

$$Y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} x_0 +$$

$$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} x_2 +$$

$$\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} x_1$$

$$Y = \frac{(x-2)(x-3)(2)}{(1-2)(1-3)} + \frac{(x-1)(x-3)(-1)}{(2-1)(2-3)}$$

$$+ \frac{(x-1)(x-2)2}{(3-1)(3-2)}$$

$$x = \frac{1}{x-1} + \frac{1}{x-2} + \frac{1}{x-3}$$

Brain Spot