Page No. ____ __/__/__

# Properties of gases

1 Prove that $C_p - C_v = R$ with usual notation.

Let, $T_1$ is the initial temperature of gas and $V_1$ is the initial specific volume of gas

$T_2$ is the Final temperature and $V_2$ is Final volume of gas.

$P$ is the pressure and $R$ is specific gas constant of gas.

$C_p$ is Specific Pressure constant of gas and $C_v$ is specific Volume Constant of gas;

Here, 1 kg heat Suppied at Constant volume $= C_v(T_2 - T_1)$

Temperature is incrase From $T_1$ to $T_2$.

Constant volume incrase internal Energy of gas but not external work dome,

So, Change in internal Energy $du = C_v(T_2 - T_1)$ — (1)

1 kg heat Suppied at Constant pressure and temperature is incrase From $T_1$ to $T_2 = C_p(T_2 - T_1)$ — (2)

1) Increse in internal Energy of gas $= C_v(T_2 - T_1)$

2) Energy to overcome extranal resistance

$$= P(V_2 - V_1) \quad — \quad 3$$

From eq$^n$ 1, 2 and 3 we can write,

$$\therefore C_p(T_2 - T_1) = C_v(T_2 - T_1) + P(V_2 - V_1)$$

$$\therefore P(V_2 - V_1) = C_p(T_2 - T_1) - C_v(T_2 - T_1) \quad — \quad 4$$

We know that, $PV_1 = RT_1$ and $PV_2 = RT_2$

$$\therefore P(V_2 - V_1) = R(T_2 - T_1)$$

We put eq$^n$ no. 4 value,

$$\therefore C_p(T_2 - T_1) - C_v(T_2 - T_1) = R(T_2 - T_1)$$

$$\therefore C_p - C_v = R$$

Page No. ___ ___/___/___

2 Derive following expression for polytropic process with usual notations.

a) $\delta Q = \frac{\gamma - n}{\gamma - 1} \times$ Work done

b) $\delta U = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$

a $\delta Q = \frac{\gamma - n}{\gamma - 1} \times$ Work done



In Polytropic Process, $\Delta U = mC_v(T_2 - T_1)$ and

$$W = \frac{P_1V_1 - P_2V_2}{n-1}$$

We know that, In this Process Heat Transfer is $Q = \Delta U + W$

$$\therefore Q = mC_v(T_2 - T_1) + \frac{P_1V_1 - P_2V_2}{n-1}$$

now, $C_v = \frac{R}{\gamma - 1}$

$$\therefore Q = m \cdot \frac{R}{\gamma - 1} \cdot (T_2 - T_1) + \frac{P_1V_1 - P_2V_2}{n-1} \quad 1)$$

We know that $P_1V_1 = mR_1$ and $P_2V_2 = mR_2$

$$\therefore Q = \frac{P_2V_2 - P_1V_1}{\gamma - 1} \cdot (T_2 - T_1) + \frac{P_2V_2 - P_1V_1}{n-1}$$

$$\therefore Q = P_1V_1 - P_2V_2 \left[ \frac{-1}{\gamma - 1} + \frac{1}{n-1} \right]$$

$$\therefore Q = P_1V_1 - P_2V_2 \left[ \frac{\gamma - 1 - n + 1}{(n-1)(\gamma - 1)} \right]$$

$$\therefore Q = \frac{P_1V_1 - P_2V_2}{n-1} \cdot \frac{\gamma - n}{\gamma - 1}$$

$$\therefore Q = \frac{\gamma - n}{\gamma - 1} \times W$$



b $\delta U = \frac{P_2V_2 - P_1V_1}{\gamma - 1}$

In this Process $\Delta U = U_2 - U_1$

$= -m_1 C_v (T_2 - T_1) + m_2 C_v (T_2 - T_1)$

$$\therefore \Delta U = -m_1 C_v (T_2 - T_1) + m_2 C_v (T_2 - T_1)$$

Now $C_v = \frac{R}{\gamma - 1}$

$$\therefore \Delta U = -m_1 \frac{R_1}{\gamma - 1} (T_2 - T_1) + m_2 \frac{R_2}{\gamma - 1} (T_2 - T_1)$$

$$\therefore \Delta U = \frac{P_2 V_2}{\gamma - 1} - \frac{P_1 V_1}{\gamma - 1}$$

$$\therefore \Delta U = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1}$$

3 Write about Combined gas law.

In practice pressure, volume and temperature of gas may change equaly. In this Condication we can not applied one law.

That's we combined equation for such cases.

From Boyle's law, $V \propto \frac{1}{P}$ Here, T is Constant.

From Charle's law, $V \propto T$

Here, P is Constant.

So, We can write that,

$$V \propto \frac{T}{P}$$

$$\therefore V = \frac{CT}{P}$$ Here, C is Constant.

$$\therefore \frac{PV}{T} = C$$

Page No. ____
___/___/___

If gas change in 1 to 2 state. So properties From $P_1$, $V_1$, $T_1$ to $P_2$, $V_2$, $T_2$

We can write, $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = C$

The constant $C$ is value of $mR$.

Here, $m$ = mass of gas
$R$ = Specific gas constant

$$\therefore PV = mRT$$

This equation know as "Characteristic gas equation".



4) For adiabatic Process prove that $p \cdot V^{\gamma} = C$

For non-flow Process,

$$\delta Q = dU + \delta W$$

$$\delta Q = C_v dT + p \cdot dv$$

For adiabatic Process $\delta Q = 0$

$$\therefore C_v \cdot dT + p \cdot dv = 0 \quad \text{— (1)}$$

Page No. ___
__/__/__

We know that $PV = RT$ than we can write that $p \cdot dv + V \cdot dp = R \cdot dT$

$$\therefore dT = \frac{P \cdot dv + V \cdot dP}{R}$$

We put $dT$ value in equation 1.

$$\therefore C_v \cdot \frac{P \cdot dv + V \cdot dP}{R} + P \cdot dv = 0$$

$$\therefore C_v (P \cdot dv + V \cdot dP) + R P \cdot dv = 0$$

$$\therefore C_v \cdot P \cdot dv + C_v \cdot V dp + (C_p - C_v) \cdot P \cdot dv = 0$$

$$\therefore C_v \cdot P \cdot dv + C_v \cdot V \cdot dp + C_p \cdot P \cdot dv - C_v \cdot P \cdot dv = 0$$

$$\therefore C_v \cdot V \cdot dp + C_p \cdot P dv = 0$$

divide this $eq^n$ by $PV \times C_v$

$$\therefore \frac{C_v \cdot V dP}{PV \cdot C_v} + \frac{C_p \cdot P \cdot dv}{PV \cdot C_v} = 0$$

$$\therefore \frac{dP}{P} + \frac{C_p}{C_v} \cdot \frac{dV}{V} = 0 C$$

$$\therefore \gamma \cdot \frac{dV}{V} + \frac{dP}{P} = 0 C$$

$$\therefore \gamma \cdot \ln V + \ln P = \ln C$$

$$\therefore V^{\gamma} \cdot P = C$$

5 Derive expression for polytropic index "n".

- From Boyle's law, $V \propto \frac{1}{P}$ Here, T is constant

$$\therefore V = \frac{C}{P}$$ Here, C is constant of proportionality

$$\therefore PV = C$$

- From law of Polytropic Process,

$$PV^n = C$$

If gas change state 1 to 2 the value will change $P_1 V_1$ to $P_2 V_2$

We can write that, $P_1 V_1^n = P_2 V_2^n$

$$\therefore \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n$$

We take logrithm both side,

$$\therefore \ln \frac{P_1}{P_2} = n \cdot \ln \frac{V_2}{V_1}$$

$$\therefore n = \frac{\ln P_1/P_2}{\ln V_2/V_1}$$