Unit : 7 : Solution of non-linear and Linear Equation

* Task : 1 : Bisection or Bolzano's Method

1 Find a real root of the equation $x^3 - 4x - 9 = 0$ using the bisection method correct upto 4 decimal place.

=> $F(x) = x^3 - 4x - 9$

For Finding the roots we have to select a and b,

$F(0) = (0)^3 - 4(0) - 9 = 0 - 9 < 0$

$F(1) = (1)^3 - 4(1) - 9 = -12 < 0$

$F(-1) = (-1)^3 - 4(-1) - 9 = -6 < 0$

$F(2) = (2)^3 - 4(2) - 9 = -9 < 0$

$F(-2) = (-2)^3 - 4(2) - 9 = -9 < 0$

$F(2.5) = (2.5)^3 - 4(2.5) - 9 = -3.375 < 0$

$F(3) = (3)^3 - 4(3) - 9 = 6 > 0$

For Bisection Method.

Here $a = 3$ and $b = 2.5$ then curves lies between a and b.

$\therefore x_1 = \dfrac{a+b}{2} = \dfrac{3+2.5}{2} = 2.75$

$F(2.75) = 0.96 > 0$

Here, $a = 2.75$ and $b = 2.5$ then curves lies between a and b.

$\therefore x_2 = \dfrac{2.5 + 2.75}{2} = 2.625$

$\therefore f(2.625) = -1.412 < 0$

Here, Take $a = 2.625$ and $b = 2.75$ then curves lies between a and b.

$\therefore x_3 = \dfrac{2.625 + 2.75}{2} = 2.6875$

$\therefore F(2.6875) = -0.334 < 0$

Select $a = 2.75$ and $b = 2.6875$ So,
Curves lies between a and b.

$\therefore x_4 = \frac{2.75 + 2.6875}{2} = 2.71875$

$\therefore f(2.71875) = 0.2209 \cancel{<0} > 0$

Here, Curves lies between 2.71875 and 2.6875.

$\therefore \cancel{f(2.68}$

$\therefore x_5 = \frac{2.71875 + 2.6875}{2} = 2.70312$

$\therefore f(2.70312) = -0.06107 < 0$

Here, Curves lies between 2.70312 and 2.71875

$\therefore x_6 = \frac{2.70312 + 2.71875}{2} = 2.710937$

$\therefore f(2.710937) = 0.07936 > 0$

Here, Curves lies between x_6 and x_5

$x_7 = \frac{2.710937 + 2.70312}{2} = 2.707031$

$\therefore F(x_7) = 8.044 \times 10^{-3} > 0$

Here, Curves lies between x_7 and x_5

$$x_8 = \frac{2.707031 + 2.70312}{2} = 2.70507$$

$\therefore F(x_8) = -0.0260 < 0$

Here, Curves lies between x_8 and x_7

$$\therefore x_9 = \frac{2.70507 + 2.707031}{2} = 2.70605$$

$\therefore F(x_9) = 8.589 \times 10^{-3} > 0$

Here, Curves lies between x_9 and x_7

$$x_{10} = \frac{2.70605 + 2.707031}{2} = 2.70654$$

$F(x_{10}) = 2.765 \times 10^{-4} < 0$

Here, Curves lies between x_{10} and x_9

SMVS
Page No.
Date : / /

$x_{11} = \frac{2.70654 + 2.70605}{2} = 2.70629$

$f(x_{11}) = -4.2769 < 0$

Here, Curves lies between 2.70629 and 2.70605

$x_{12} = \frac{2.70605 + 2.70629}{2} = 2.706420$

$f(x_{12}) = -1.940 \times 10^{-3} < 0$

Here, Curves lies between 2.706420 and 2.70605

$x_{13} = \frac{2.706420 + 2.70605}{2} = 2.706481$

$f(x_{13}) = -8.440 \times 10^{-4} < 0$

=> Here, we get 4 Decimal Root for $f(x)$.

Root of $f(x)$ is 2.7064.

2 Using the bisection method to obtain the root of the equation $f(x) = x^3 - x - 1$ correct up to four decimal place.

=> Let $F(x) = x^3 - x - 1$

For Bisection method, we have to select a and b.

$F(0) = -1$	$F(1) = 1$
$F(-1) = -1$	$F(2) = 5$
$F(-2) = -7$	$F(3) = 23$
$F(-3) = -25$	$F(1.5) = 0.875$

Here, we select 1 and 1.5 as a a and b.

So, Curves lies between 1 and 1.5.

$x_1 = \frac{1.5+1}{2} = 1.25$ — 0

$F(x_1) = -0.29 < 0$.

So, Curves lies between 1.25 and 1.5

$\therefore x_2 = \frac{1.25+1.5}{2} = 1.375$

$\therefore F(x_2) = 0.2246 > 0$

So, Curves lies between 1.25 and 1.375.

$\therefore x_3 = \frac{1.375 + 1.25}{2} = 1.3125$

$\therefore F(x_3) = -0.0515 < 0$

So, Curves lies between 1.375 and 1.3125.

$\therefore x_4 = \frac{1.375 + 1.3125}{2} = 1.34375$

$\therefore F(x_4) = 0.0826 > 0$

So, Curves lies between x_4 and x_3.

$\therefore x_5 = \frac{1.3125 + \cancel{0.0}\ 1.34375}{2} = 1.3281$

$\therefore F(x_5) = 0.01457 \cancel{<0} > 0$

So, Curves lies between 1.3125 and 1.3281

$x_6 = \frac{1.3125 + 1.3281}{2} = 1.3203125$

$F(x_6) = -0.01871 < 0$

So, Curves lies between,

$x_7 = \frac{1.3281 + 1.32031}{2} = 1.32421$

$\therefore F(x_7) = -2.13 \times 10^{-3} < 0$

So, Curves lies between, 1.32421 and 1.3281

$x_8 = \frac{1.3281 + 1.32421}{2} = 1.32616$

$\therefore F(x_8) = 6.18 \times 10^{-3} > 0$

So, Curves lies between 1.32616 and 1.32421.

$x_9 = \frac{1.32421 + 1.32616}{2} = 1.32519$

$\therefore F(x_9) = 2.025 \times 10^{-3} > 0$

So, Curves lies between 1.3261 and 1.3242.

$x_{10} = \frac{1.3261 + 1.3242}{2} = 1.32470$

SMVS
Page No.
Date: / /

$\therefore \cancel{f(x)}_0$

$\therefore f(x_{10}) = -5.52 \times 10^{-5} < 0$

So, Curves lies between 1.3247 and 1.32519

$\therefore x_{11} = \frac{1.3249 + 1.32519}{2} = 1.324948$

$\therefore f(x_{11}) = 9.47 \times 10^{-4} > 0$

So, Curves lies between 1.324948 and 1.32470

$\therefore x_{12} = \frac{1.324948 + 1.32470}{2} = \cancel{1.324948}$ 1.324826

$\therefore f(x_{12}) = \cancel{9}\ 0.013 > 0$

So, Curves lies between 1.324826 and 1.32470

$\therefore x_{13} = \frac{1.324826 + 1.32470}{2} = 1.324765$

$\therefore f(x_{13}) = 2 \times 10^{4} \cancel{x} > 0$

So, Curves lies between 1.324765 and 1.32470

$$x_{14} = \frac{1.324765 + 1.32470}{2} = 1.32473$$

$$f(x_{14}) = 7.26 \times 10^{-5} > 0$$

=> Here, we get 4 Decimal Root of F(x).

Hence, Root of F(x) is 1.3247

3 Find the positive root of $x - \cos(x) = 0$ correct upto three decimal places by bisection Method.

=> Let $F(x) = \cos(x) - x$

For Bisection method, we have to select a and b.

$F(0) = 1 > 0$, $f(1) = -0.45 < 0$

Here, we select 1 and 0 as a a and b.

$$\therefore x_1 = \frac{1+0}{2} = 0.5$$

$$\therefore f(0.5) = \cancel{0.499}\ 0.377 > 0$$

So, Curves lies between 0.5 and 1.

$\therefore x_2 = \frac{1+0.5}{2} = 0.75$

$\therefore f(0.75) = -0.018 \cancel{>0} < 0$

So, Curves lies between 0.75 and 0.5.

$\therefore x_3 = \frac{0.75+0.5}{2} = 0.625$

$\therefore f(0.625) = 0.1859 > 0.$

So, Curves lies between 0.625 and 0.75.

$\therefore x_4 = \frac{0.625+0.75}{2} = 0.6875$

$\therefore f(x_4) = 0.0853 > 0.$

So, Curve lies between 0.6875 and 0.75.

$\therefore x_5 = \frac{0.75+0.6875}{2} = 0.71875$

$\therefore f(x_5) = 0.0338 > 0.$

Page No.
Date : / /

So, Curve lies between 0.71875 and 0.75

$$x_6 = \frac{0.75 + 0.71875}{2} = 0.73437$$

$$f(x_6) = 7.88 \times 10^{-3} > 0.$$

So, Curve lies between 0.73437 and 0.75.

$$\therefore x_7 = \frac{0.73437 + 0.75}{2} = 0.7421$$

$$\therefore f(x_7) = -5.04 \times 10^{-3} < 0$$

So, Curve lies between 0.7421 and 0.73437.

$$\therefore x_8 = \frac{0.7421 + 0.73437}{2} = 0.7382$$

$$\therefore f(x_8) = 1.48 \times 10^{-3} > 0.$$

So, Curve lies between 0.7382 and 0.7421

$$\therefore x_9 = \frac{0.7382 + 0.7421}{2} = 0.74015$$

$$\therefore f(x_9) = -1.78 \times 10^{-3} < 0$$

So, Curve lies between 0.74015 and 0.7382.

$$x_{10} = \frac{0.74015 + 0.7382}{2} = 0.73917$$

$\therefore f(x_{10}) = -1.42 \times 10^{-4}$ ~~>0~~ <0

So, Curve lies between 0.73917 and 0.7382.

$$\therefore x_{11} = \frac{0.73917 + 0.7382}{2} = 0.73868$$

$\therefore f(x_{11}) = 6.69 \times 10^{-4} > 0$

So, Curve lies between 0.73868 and 0.73917

$$\therefore x_{12} = \frac{0.73868 + 0.73917}{2}$$

$$x_{12} = 0.7389$$

→ Here, we get 3 decimal root of $f(x)$.

So, Root of $f(x)$ is 0.738

4 Using the bisection method to obtain the root of the equation $F(x) = Cos(x) - xe^{(x)} = 0$ correct upto two decimal place.

=> Let $F(x) = Cos(x) - xe^{x}$

For Bisection method, we have to select a and b.

$F(0) = 1 > 0$ $\quad$ $F(-1) = 0.90$

$F(1) = -2.71 < 0$

Here, Curves lies between 0 and 1.

$\therefore x_1 = \frac{0+1}{2} = 0.5$

$\therefore F(x_1) = 0.0532 > 0$.

So, Curve lies between 0.5 and 1.

$\therefore x_2 = \frac{1+0.5}{2} = 0.75$.

$\therefore F(x_2) = -0.856 < 0$

So, Curve lies between 0.75 and 0.5

$$\therefore x_3 = \frac{0.75+0.5}{2} = +0.\overset{625}{\cancel{356<0}}$$

$$\therefore f(x_3) = -0.356 < 0.$$

So, Curve lies between 0.625 and 0.5.

$$\therefore x_4 = \frac{0.5+0.625}{2} = 0.5625$$

$$\therefore f(x_4) = -0.141 < 0.$$

So, Curve lies between 0.5625 and 0.5

$$\therefore x_5 = \frac{0.5+0.5625}{2} = 0.53125.$$

$$\therefore f(x_5) = -0.0415 < 0.$$

So, Curve lies between 0.53125 and 0.5.

$$\therefore x_6 = \frac{0.5+0.53125}{2} = 0.51562$$

$$\therefore f(x_6) = 6.49 \times 10^{-3} > 0$$

So, Curve lies between 0.53125 and 0.51562

$$\therefore x_7 = \frac{0.53125+0.51562}{2} = 0.5234$$

$$\therefore f(x_7) = -0.0172 < 0$$

So, Curve lies between 0.5234 and 0.51562.

$$\therefore x_8 = \frac{0.5234+0.51562}{2} = 0.51951$$

$$\therefore f(x_8) = -5.33 \times 10^{-3} < 0$$

So, Curve lies between 0.51951 and 0.51562.

$$\therefore x_9 = \frac{0.51951+0.51562}{2} = 0.517$$

$$\therefore f(x_9) = 2.30 \times 10^{-3}$$

→ Here, we get 2 Decimal root of f(x).

Root of f(x) is 0.51.

5 Find a real root of the eqution $x^3 - 5x + 3 = 0$ using the bisection method correct upto 4 Decimal place.

$\Rightarrow$ Let $F(x) = x^3 - 5x + 3$

For Bisection Method, we have to select a and b.

$F(0) = 3 > 0$, $f(1) = -1 < 0$

$\therefore x_1 = \frac{0+1}{2} = 0.5$

$F(0.5) = 0.625 > 0.$

Here; Curve lies between 0.5 and 1.

$\therefore x_2 = \frac{1+0.5}{2} = 0.75$

$\therefore F(0.75) = -0.3281 < 0$

So, Curve lies between 0.75 and 0.5.

$\therefore x_3 = \frac{0.75+0.5}{2} = 0.625$

$\therefore F(0.625) = 0.1191 > 0.$

So, Curve lies between 0.625 and 0.75.

$$\therefore x_4 = \frac{0.625+0.75}{2} = 0.6875$$

$$\therefore f(x_4) = -0.1125 < 0.$$

So, Curve lies between 0.625 and 0.6875.

$$\therefore x_5 = \frac{0.625+0.6875}{2} = 0.65625$$

$$\therefore f(x_5) = 1.37 \times 10^{-3} > 0.$$

So, Curve lies between 0.65625 and 0.6875.

$$\therefore x_6 = \frac{0.65625+0.6875}{2} = 0.67187$$

$$\therefore f(x_6) = -0.056 < 0.$$

So, Curve lies between 0.67187 and 0.65625.

$$\therefore x_7 = \frac{0.67187+0.65625}{2} = 0.66406$$

$$\therefore f(x_7) = -0.0274 < 0$$

Page No.
Date : / /

Curves lies between 0.65625 and 0.66406.

$$\therefore x_8 = \frac{0.65625 + 0.66406}{2} = 0.66015$$

$$\therefore f(x_8) = -0.0130 < 0.$$

So, Curves lies between 0.65625 and 0.66015.

$$\therefore x_9 = \frac{0.66015 + 0.65625}{2} = 0.6582$$

$$\therefore f(x_9) = -5.84 \times 10^{-3} < 0.$$

So, Curves lies between 0.65625 and 0.6582.

$$\therefore x_{10} = \frac{0.6582 + 0.65625}{2} = \cancel{-2.22 \times 10^{-3}} = 0.65722$$

$$\therefore f(x_{10}) = -2.22 \times 10^{-3} < 0.$$

So, Curves lies between 0.65722 and 0.65625.

$$\therefore x_{11} = \frac{0.65625 + 0.65722}{2} = 0.65673$$

$$\therefore f(x_{11}) = -4.06 \times 10^{-4} < 0.$$

So, Curves lies between 0.65625 and 0.65673

$\therefore x_{12} = \frac{0.65625 + 0.65673}{2} = 0.65649$

$\therefore f(x_{12}) = 4.83 \times 10^{-4} > 0$

So, Curves lies between 0.65649 and 0.65673.

$\therefore x_{13} = \frac{0.65673 + 0.65649}{2} = 0.65661$

$\therefore f(x_{13}) = 3.86 \times 10^{-5} > 0.$

So, Curves lies between 0.65661 and 0.65673.

$\therefore x_{14} = \frac{0.65673 + 0.65661}{2} = 0.65667$

$\therefore f(x_{14}) = -1.83 \times 10^{-4}.$

$\Rightarrow$ Here, we get 4 Decimal Root of f(x).

Root of f(x) is, 0.6566.

Page No.
Date : / /

* Task : 2 Regula Falsi Method and Secant Method

1 Compute the real root of $F(x) = x - 2\sin(x) = 0$ correct up to 6 decimal places using Regula Falsi method.

Let $F(x) = x - 2\sin(x) = 0$
For Regula Falsi method
we have to select a and b.

So, $F(0) = 0$
$F(1) = -0.6829 < 0$
$F(2) = 0.181 > 0$

For Regula Falsi method $a = 1$ and $b = 2$.

$$\therefore x_1 = \frac{1F(2) - 2F(1)}{F(2) - F(1)}$$

$$= 1.790485$$

$\therefore F(x_1) = -0.1614 < 0$

Root lies between x_1 and 2.

$$\therefore x_2 = \frac{x_1 F(2) - 2F(x_1)}{F(2) - F(x_1)}$$

1.88924586

$\therefore f(x_2) = -0.010198 < 0$

Root lies between $f(x_2$ and 2.

$$\therefore x_3 = \frac{x_2 f(2) - 2f(x_2)}{f(2) - f(x_2)}$$

$$= 1.8951531$$

$\therefore f(x_3) = -0.000558 < 0$

Root lies between x_3 and 2.

$$\therefore x_4 = \frac{x_3 f(2) - 2f(x_3)}{f(2) - f(x_3)}$$

$$= 1.8954753$$

$\therefore f(x_4) = -0.0000031 < 0$

Root lies between x_4 and 2.

$$\therefore x_5 = \frac{x_4 f(2) - 2f(x_4)}{f(2) - f(x_4)}$$

$$= 1.895493199$$

$\therefore f(x_5) = -0.00000174 < 0$

Root lies between x_5 and 2.

$$\therefore x_6 = \frac{x_5 f(2) - 2 f(x_5)}{f(2) - f(x_5)}$$

$$= 1.8954942$$

$\therefore f(x_6) = -0.0000001038 < 0$

Root lies between x_6 and 2.

$$\therefore x_7 = \frac{x_6 f(2) - 2 f(x_6)}{f(2) - f(x_6)}$$

$$= 1.8954942$$

Here, we get six decimal
Root 1.895494.

2 Slove $xe^x - 1 = 0$ correct upto three decimal place by using Regula Falsi method.

Let $F(x) = xe^x - 1$
For Regula Falsi method we have to select a and b.

So, $f(0) = -1 < 0$
$f(1) = 1.7182 > 0$

For Regula Falsi method
$a = 1, \quad b = 0.$

$$x_0 = \frac{0 f(1) - 1 f(0)}{f(1) - f(0)}$$

$= 0.36790$

$\therefore f(x_0) = -0.4684 < 0$

Root lies between x_0 and a.

$$x_1 = \frac{x_0 f(1) - 1 f(x_0)}{f(1) - f(x_0)}$$

$= 0.5033$

$\therefore f(x_1) = -0.1674 < 0$

Root lies between x_1 and a.

$$\therefore x_2 = \frac{x_1 f(1) - 1 f(x_1)}{f(1) - f(x_1)}$$

$= 0.5473$

$\therefore f(x_2) = -0.05376 < 0$

Root lies between x_2 and a.

$$\therefore x_3 = \frac{x_2 f(1) - 1 f(x_2)}{f(1) - f(x_2)}$$

$$= 0.5610$$

$\therefore f(x_3) = -0.0167 < 0$

Root lies between x_3 and a.

$$\therefore x_4 = \frac{x_3 f(1) - 1 f(x_3)}{f(1) - f(x_3)}$$

$$= 0.5652$$

$\therefore f(x_4) = -0.005361 < 0$

Root lies between x_4 and a.

$$\therefore x_5 = \frac{x_4 f(1) - 1 f(x_4)}{f(1) - f(x_4)}$$

$$= 0.5665$$

$\therefore f(x_5) = -0.00177 < 0$

Root lies between x_5 and a.

$$\therefore x_6 = \frac{x_5 f(1) - 1 f(x_5)}{f(1) - f(x_5)}$$

$$= 0.566$$

Here, we get three decimal root 0.566.

3 Find the approximate root of $x^3 - 2x - 1 = 0$, using Regula Falsi method correct upto Three decimal.

Let $F(x) = x^3 - 2x - 1 = 0$
For Regula Falsi method we have to select a and b.

So, $F(0) = -1$
$F(1) = -2 < 0$
$F(2) = 3 > 0$

For Regula Falsi method
$a = 1, b = 2$.

$$x_0 = \frac{1F(2) - 2F(1)}{F(2) - F(1)}$$

$= 1.4$

$F(x_0) = -1.056 < 0$.
$F(2) = 3 > 0$

$$\therefore x_1 = \frac{x_0 F(2) - 2F(x_0)}{F(2) - F(x_0)}$$

$$= 1.5562$$

$\therefore F(x_2) = 4.029 > 0$
$\therefore F(x_1) = -0.3435 < 0$

$$\therefore x_3 = \frac{x_1 F(x_2) - x_2 F(x_1)}{F(x_2) - F(x_1)}$$

$$= 1.59870$$

$\therefore F(x_3) = -0.1113 < 0$
$\therefore F(x_2) = 4.029 > 0$

$$\therefore x_4 = \frac{x_3 F(x_2) - x_2 F(x_3)}{F(x_2) - F(x_3)}$$

$$= 1.61210$$

$\therefore F(x_4) = -0.0345 < 0$
$\therefore F(x_2) = 4.029 > 0$

$$\therefore x_5 = \frac{x_4 F(x_2) - x_2 F(x_4)}{F(x_2) - F(x_4)}$$

$$= 1.61617$$

$\therefore F(x_5) = -0.01086 < 0$
$\therefore F(x_2) = 4.029 > 0$

$\therefore x_6 = \dfrac{x_5 f(x_2) - x_2 f(x_5)}{f(x_2) - f(x_5)}$

$= 1.61746$

$\therefore f(x_6) = -0.00334 < 0$

$\therefore f(x_2) = 4.029 > 0$

$\therefore x_7 = \dfrac{x_6 f(x_2) - x_2 f(x_6)}{f(x_2) - f(x_6)}$

$= 1.6178$

Here, we get three decimal root 1.617.

4 Find the root of $x \log_{10} x - 1.9 = 0$ correct up to three decimal places with $x_0 = 3$ and $x_1 = 4$

Let $f(x) = x \log_{10} x - 1.9$

Here Given $a = 3$ and $b = 4$.

So, $f(x_0) = -0.4686 < 0$

$f(x_1) = 0.5082 > 0$

SMVS
Page No.
Date: / /

$$\therefore x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= 3.4797$$

$\therefore f(x_2) = -0.015596 < 0$

$\therefore f(x_1) = 0.5082 > 0$

$$\therefore x_3 = \frac{x_2 f(x_1) - 4 f(x_2)}{f(x_1) - f(x_2)}$$

$$= 3.49519$$

$\therefore f(x_3) = -0.000464 < 0$

$\therefore f(x_1) = 0.5082 > 0$

$$\therefore x_4 = \frac{x_3 f(x_1) - 4 f(x_3)}{f(x_1) - f(x_3)}$$

$$= 3.4956$$

Here, we get Three decimal root 3.4956.

5 Find the root of $\cos x - xe^{x} = 0$ correct upto three decimal places.

Page No.
Date: / /

Let $F(x) = \cos x - xe^{x}$
For regula Falsi method
we have to select a and b.

So, $F(0) = 1 > 0$
$F(1) = -2.177 < 0$

$\therefore x_1 = \dfrac{1F(0) - 0F(1)}{F(0) - F(1)}$

$= 0.3147$

$\therefore F(x_1) = 0.51966 > 0$
$\therefore F(1) = -2.177 < 0$

$\therefore x_2 = \dfrac{1F(x_1) - x_1F(1)}{F(x_1) - F(1)}$

$= 0.44676$

$\therefore F(x_2) = 0.2034 > 0$
$\therefore F(1) = -2.177 < 0$

$\therefore x_3 = \dfrac{1F(x_2) - x_2F(1)}{F(x_2) - F(1)}$

$= 0.49403$

$\therefore F(x_3) = 0.07075 > 0$
$\therefore F(1) = -2.177 < 0$

$$\therefore x_4 = \frac{1\,F(x_3) - x_3 F(1)}{F(x_3) - F(1)}$$

$$= 0.50995$$

$\therefore F(x_4) = 0.02357 > 0$

$\therefore F(1) = -2.177 < 0$

$$\therefore x_5 = \frac{1\,F(x_4) - x_4 F(1)}{F(x_4) - F(1)}$$

$$= 0.51536$$

$\therefore F(x_5) = 0.007275 > 0$

$\therefore F(1) = -2.177 < 0$

$$\therefore x_6 = \frac{1\,F(x_5) - x_5 F(1)}{F(x_5) - F(1)}$$

$$= 0.51697$$

$\therefore F(x_6) = 0.00238 > 0$

$\therefore F(1) = -2.177 < 0$

$$\therefore x_7 = \frac{1\,F(x_6) - x_6 F(1)}{F(x_6) - F(1)}$$

$$= 0.51749$$

$F(x_7) = 0.000790 > 0$
$F(1) = -2.177 < 0$

$$\therefore x_8 = \frac{1\,F(x_8) - x_8\,F(1)}{F(x_8) - F(1)}$$

$$= 0.517$$

Here, we get three decimal root 0.517.

* Task : 3 Newton-Raphson Method

1 Compute the real root of $F(x) = x^4 - x^3 + 10x + 7$ correct upto three decimal places between -2 and -1.

Let, Given $F(x) = x^4 - x^3 + 10x + 7$

$\therefore F'(x) = 4x^3 - 3x^2 + 10$

Here, Given $a = -2$ and $b = -1$.

So, $x_0 = \frac{a+b}{2} = \frac{-2-1}{2} = -1.5$

For N-R Method,

$\therefore F(x_0) = 0.4375$

$\therefore F'(x_0) = -10.25$

$\therefore x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$

$= -1.4573$

$\therefore F(x_1) = 0.03210$

$F'(x_1) = -8.750$

$\therefore x_2 = x_1 - \dfrac{F(x_1)}{F'(x_1)}$

$= -1.4536$

$\therefore F(x_2) = -4.136 \times 10^{-5}$
$F'(x_2) = -8.6244$

$\therefore x_3 = x_2 - \dfrac{F(x_2)}{F'(x_2)}$

$= -1.4536$

Here, we get Four decimal root -1.4536 of $F(x)$.

2 Solve $x^3 + x - 1 = 0$ Correct upto 6 decimal places by ~~Ne~~ N-R method.

Let Given $F(x) = x^3 + x - 1$
$F'(x) = 3x^2 + 1$

For N-R method, we have to Find a and b.

$\therefore F(a) = F(0) = -1$
$\therefore F(b) = F(1) = 1$

So, $x_0 = \frac{a+b}{2} = \frac{1}{2} = 0.5$

Using N-R method;

$\therefore F(x_0) = -0.375 \quad F'(x_0) = 1.75$

$\therefore x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$

$= 0.71428$

$\therefore F(x_1) = 0.07870 \quad F'(x_1) = 2.5305$

$\therefore x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$

$= 0.68317$

$\therefore F(x_2) = 2.01 \times 10^{-3} \quad F'(x_2) = 2.4001$

$\therefore x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$

$= 0.682339$

$\therefore F(x_3) = 2.6 \times 10^{-5} \quad F'(x_3) = 2.3767$

$$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.682327$$

$\therefore$ ~~$x_5 = x_4$~~

$\therefore F(x_4) = -1.9 \times 10^{-6}$ $F'(x_4) = 2.3967$

$$\therefore x_5 = x_4 - \frac{F(x_4)}{F'(x_4)}$$

$$= 0.682327$$

Here, we get six decimal root 0.682327 of $F(x)$.

3 Find the root of $F(x) = x - \cos x$ correct upto three decimal places.

Let Given $F(x) = x - \cos x$

$F'(x) = 1 + \sin x$

For N-R method we have to Find a and b.

$\therefore F(a) = F(0) = -1$

$\therefore F'(b) = F(1) = 0.4596$

Now, $x_0 = \frac{a+b}{2} = \frac{1}{2} = 0.5$

$F(x_0) = -0.3775$ $\quad F'(x_0) = 1.4794$

Using N-R method,

$\therefore x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$

$= 1$

$\therefore F(x_1) = 0.4596$ $\quad F'(x_1) = 1.8414$

$\therefore x_2 = x_1 - \frac{F(x_1)}{F'(x_1)}$

$= 0.75036$

$\therefore F(x_2) = 0.01891$ $\quad F'(x_2) = 1.68190$

$\therefore x_3 = x_2 - \frac{F(x_2)}{F'(x_2)}$

$= 0.739112$

$\therefore F(x_3) = 0.00004964$ $\quad F'(x_3) = 1.67363$

$$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.7391125$$

Here, we get Three decimal root of 0.739112 .

4 Find the root of $x = e^{-x}$ correct upto four decimal and start with $x_0 = 0.6$

Let Given $F(x) = x - e^{-x}$
$F'(x) = 1 + e^{-x}$

Here, Given First $x_0 = 0.6$

$\therefore F(x_0) = 0.0511 \quad F'(x_0) = 1.5488$

Using N-R method,

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)}$$

$$= 0.5671$$

$\therefore F(x_1) = -6.7 \times 10^{-5} \quad F'(x_1) = 1.5671$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.5587$

$\therefore f(x_2) = -0.0132 \quad f'(x_2) = 1.5719$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.56714$

$\therefore f(x_3) = -5.15 \times 10^{-6} \quad f'(x_3) = 1.5671$

$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 0.58853$

$\therefore f(x_4) = 0.0338 \quad f'(x_4) = 1.5551$

$\therefore x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$

$= 0.58853$

Here, we get four decimal root 0.5885 of $f(x)$.

5 Using N-R method Find an iterative formula to Find $\sqrt{N}$ and hence Find $\sqrt{5}$.

Let Given $x = \sqrt{N}$

$\therefore x^2 = N$

$\therefore x^2 - N = 0$

Here, We have $N = 5$.

So, $F(x) = x^2 - 5$

$F'(x) = 2x$

For N-R method we have to Find a and b.

Here, $2 < \sqrt{5} < 3$

So, $a = 2$ and $b = 3$

So, $F(a) = F(2) = -1$

$F(b) = F(3) = 4$

Using N-R method,

$\therefore x_0 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$

$\therefore F(x_0) = 1.25 \quad F'(x_0) = 5$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2.4166$$

$\therefore f(x_1) = 0.8399 \quad f'(x_1) = 4.8332$

$$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.2428$$

$\therefore f(x_2) = 0.03015 \quad f'(x_2) = 4.4856$

$$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.2361$$

$\therefore f(x_3) = 1.4 \times 10^{-4} \quad f'(x_3) = 4.4722$

$$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.23606$$

Here we get three decimal value of $\sqrt{5} = 2.236$

* Task : 4 : Gauss - Jacobi Method and Gauss - Seidel Method

1 Solve the following System of equation by Gauss - Jacobi method.

$$10x + y + z = 6$$
$$x + 10y + z = 6$$
$$x + y + 10z = 6$$

Here, Given equation,

$$|10x| > |1y| + |1z|$$
$$|10y| > |1x| + |1z|$$
$$|10z| > |1x| + |1y|$$

By Gauss-Jacobi method,

$$\therefore x = \frac{1}{10}[6 - y - z]$$
$$\therefore y = \frac{1}{10}[6 - x - z]$$
$$\therefore z = \frac{1}{10}[6 - y - x]$$

For intital value of
$x = 0, y = 0, z = 0$

SMVS
Page No.
Date : / /

$\therefore x_1 = \frac{1}{10}[6] = 0.6$

$\therefore y_1 = \frac{1}{10}[6] = 0.6$

$\therefore z_1 = \frac{1}{10}[6] = 0.6$

-> For x_2, y_2 and z_2, value of
$x_1 = 0.6$, $y_1 = 0.6$, $z_1 = 0.6$

$\therefore x_2 = \frac{1}{10}[6 - 0.6 - 0.6] = 0.48$

$\therefore y_2 = \frac{1}{10}[6 - 0.6 - 0.6] = 0.48$

$\therefore z_2 = \frac{1}{10}[6 - 0.6 - 0.6] = 0.48$

-> For z_3, y_3 and x_3, value of
$x_2 = 0.48$, $y_2 = 0.48$, $z_2 = 0.48$

$\therefore x_3 = \frac{1}{10}[6 - 0.48 - 0.48] = 0.504$

$\therefore y_3 = \frac{1}{10}[6 - 0.48 - 0.48] = 0.504$

$\therefore z_3 = \frac{1}{10}[6 - 0.48 - 0.48] = 0.504$

-> For x_4, y_4 and z_4, value of
$x_3 = y_3 = z_3 = 0.504$

$\therefore x_4 = \frac{1}{10}[6 - 0.504 - 0.504] = 0.4992$

$\therefore y_4 = \frac{1}{10}[6 - 0.504 - 0.504] = 0.4992$

$\therefore z_4 = \frac{1}{10}[6 - 0.504 - 0.504] = 0.4992$

→ For x_5, y_5 and z_5, value of
$x_4 = z_4 = y_4 = 0.4992$

$\therefore x_5 = \frac{1}{10}[6 - 0.4 - 0.4] = 0.40$

$\therefore y_5 = \frac{1}{10}[6 - 0.4 - 0.4] = 0.40$

$\therefore z_5 = \frac{1}{10}[6 - 0.4 - 0.4] = 0.40$

Here, we get $x_5 = x_4$,
$y_5 = y_4$, and
$z_5 = z_4$.

So, value of $x = y = z = 0.4$

2 Solve the following system of eq^n by Gauss-Jacobi method.

$20x + 2y + z = 30$
$x - 40y - 3z = -75$
$20x - y + 10z = 30$

Here, Given equaction

$$|20x| > |2y| + |z|$$
$$|40y| > |x| + |3z|$$
$$|10z| > |2x| + |y|$$

By Gauss - Jacobi method,

$$x = \frac{1}{20}[30 - 2y - z]$$

$$y = \frac{1}{-40}[-75 - x + 3z]$$

$$z = \frac{1}{10}[30 - 2x + y]$$

-> Intial value of $x = y = z = 0$.

$$\therefore x_1 = \frac{1}{20}[30] = 1.5$$

$$\therefore y_1 = \frac{-1}{40}[-75] = 1.875$$

$$\therefore z_1 = \frac{30}{10} = 3$$

-> For x_2, y_2 and z_2, value of
$x_1 = 1.5$, $y_1 = 1.875$, $z_1 = 3$

$$\therefore x_2 = \frac{1}{20}[30 - 2(1.875) - 3]$$

$$= 1.1625$$

$$Y_2 = \frac{-1}{40}[-75 - 1.5 + 3(3)]$$
$$= 1.6875$$

$$Z_2 = \frac{1}{10}[30 - 2(1.5) + 1.875]$$
$$= 2.88875$$

→ For x_3, Y_3 and Z_3, value of
$x_2 = 1.1625$, $Y_2 = 1.6875$
$Z_2 = 2.88875$.

$$x_3 = \frac{1}{20}[30 - 2(1.6875) - 2.8875]$$
$$= 1.2393$$

$$Y_3 = \frac{-1}{40}[-75 - 1.1625 + 3(2.8875)]$$
$$= 1.6875$$

$$Z_3 = \frac{1}{10}[30 - 2(1.1625) + 1.6875]$$
$$= 2.9362$$

→ For x_4, Y_4 and Z_4, value of
$x_3 = 1.2393$, $Y_3 = 1.6875$
$Z_3 = 2.9362$

$$x_4 = \frac{1}{20}[30 - 2(1.6875) - 2.9362]$$

$= 1.1844$

$y_4 = \frac{-1}{40} [-75 - 1.2393 + 3(2.9362)]$

$= 1.6857$

$z_4 = \frac{1}{30} [30 - 2(1.2393) + 1.6875]$

$= 2.9208.$

→ For x_5, y_5 and z_5, value of $x_4 = 1.1844$, $y_4 = 1.6857$, $z_4 = 2.9208$

$x_5 = \frac{1}{20} [30 - 2(1.6857) - 2.9208]$

$= 1.1852$

$y_5 = \frac{-1}{40} [-75 - 1.1844 + 3(2.9208)]$

$= 1.6855$

$z_5 = \frac{1}{10} [30 - 2(1.1844) + 1.6857]$

$= 2.9316.$

Here, we get $x_4 = x_5$, $y_4 = y_5$ and $z_5 = z_4$

So, $x = 1.18$, $y = 1.68$, $z = 2.93.$

3 Solve the following System of equation by Gauss-Seidel method correct up to three decimal places.

$$2x + 6y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

Here Given equaction,

$$|27x| > |6y| + |-z|$$
$$|15y| > |6x| + |2z|$$
$$|54z| > |2x| + |6y|$$

By Gauss-Seidel method,

$$\therefore x = \frac{1}{27}[85 - 6y + z]$$

$$\therefore y = \frac{1}{15}[72 - 6x - 2z]$$

$$\therefore z = \frac{1}{54}[110 - 6y - 2x]$$

→ By this method,
intial value $z = 0$, $y = 0$

$$\therefore x_1 = \frac{1}{27}[85] = 3.1481$$

$\therefore y_1 = \frac{1}{15}[72 - 6(3.1481)]$

$= 3.5047$

$\therefore z_1 = \frac{1}{54}[110 - 6(3.5407) - 2(3.1481)]$

$= 1.5270$

→ For x_2, value of $z_1 = 1.5270$
$y_1 = 3.5047$

$\therefore x_2 = \frac{1}{27}[85 - 6(3.5407) + 1.5270]$

$= 2.4178$

$\therefore y_2 = \frac{1}{15}[72 - 6(2.4178) - 2(1.5270)]$

$= 3.6292$

$\therefore z_2 = \frac{1}{54}[110 - 6(3.6292) - 2(2.4178)]$

$= 1.5442$

→ For x_3, value of $z_2 = 1.5442$
$y_2 = 3.6292$

$\therefore x_3 = \frac{1}{27}[85 - 6(3.6292) + 1.5442]$

$= 2.3988$

$\therefore y_3 = \frac{1}{15}[72 - 6(2.3988) - 2(1.5442)]$

$= 3.6345$

$\therefore z_3 = \frac{1}{54}[110 - 6(3.6345) - 2(2.3988)]$

$= 1.5443.$

-) For x_4, value of $z_3 = 1.5443$
$y_3 = 3.6345$

$x_4 = \frac{1}{27}[85 - 6(3.6345) + 1.5443]$

$= 2.3976$

$y_4 = \frac{1}{15}[72 - 6(2.3976) - 2(1.5443)]$

$= 3.6350$

$z_4 = \frac{1}{54}[110 - 6(3.6350) - 2(2.3976)]$

$= 1.5443$

-) For x_5, value of $z_4 = 1.5443$
$y_4 = 3.6350$

$\therefore x_5 = \frac{1}{27}[85 - 6(3.6350) + 1.5443]$

$= 2.3975$

$$y_5 = \frac{1}{15}[72 - 6(2.3975) - 2(1.5443)]$$
$$= 3.6350$$

$$z_5 = \frac{1}{54}[110 - 6(3.6350) - 2(2.3975)]$$
$$= 1.5443$$

Here, we get three decimal value of $x = 2.397$, $y = 3.635$
$z = 1.544$

4 Solve the following system of equaction by Gauss-Seidel method.

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$2x + 2y + 10z = 14$$

Here Given equaction,

$$|10x| > |y| + |z|$$
$$|10y| > |2x| + |z|$$
$$|10z| > |2x| + |2y|$$

By Gauss-Seidel Method,

$x = \frac{1}{10} [12 - y - z]$

$y = \frac{1}{10} [13 - 2x - z]$

$z = \frac{1}{10} [14 - 2x - 2y]$

→ By this method,
initial value of $y = 0$, $z = 0$.

$x_1 = \frac{1}{10} [12] = 1.2$

$y_1 = \frac{1}{10} [13 - 2(1.2)]$

$= 1.06$

$z_1 = \frac{1}{10} [14 - 2(1.2) - 2(1.06)]$

$= 0.946$

→ For x_2, value $y_1 = 1.06$
$z_1 = 0.946$

$\therefore x_2 = \frac{1}{10} [12 - 1.04 - 0.946]$

$= 0.9994$

$y_2 = \frac{1}{10} [13 - 2(0.9994) - 0.946]$

Page No.
Date : / /

$= 1.00552$

$z_2 = \frac{1}{10}[14 - 2(0.9994) - 2(1.005)]$

$= 0.99901$

→ For x_3, value of $y_2 = 1.005$

$z_2 = 0.999$

$\therefore x_3 = \frac{1}{10}[12 - 1.005 - 0.999]$

$= 0.999$

$\therefore y_3 = \frac{1}{10}[13 - 2(0.999) - 0.999]$

$= 1.0001$

$\therefore z_3 = \frac{1}{10}[14 - 2(0.999) - 2(1.0001)]$

$= 1$

→ For x_4, value of $z_3 = 1$

$y_3 = 1$

$\therefore x_4 = \frac{1}{10}[12 - 1 - 1]$

$= 1$

$\therefore y_4 = \frac{1}{10}[13 - 2(1) - 1]$

$= 1$

$$\therefore z_4 = \frac{1}{10}[14 - 2(1) - 2(1)]$$
$$= 1$$

Here, we get $x_3 = x_4$,
$y_3 = y_4$,
$z_3 = z_4$

Value of $x = y = z = 1$

5 Solve the following system of equation by Gauss-Seidel Method.

$$100y + 130z = 230$$
$$-40x + 150y - 100z = 0$$
$$60x - 4y = 200$$